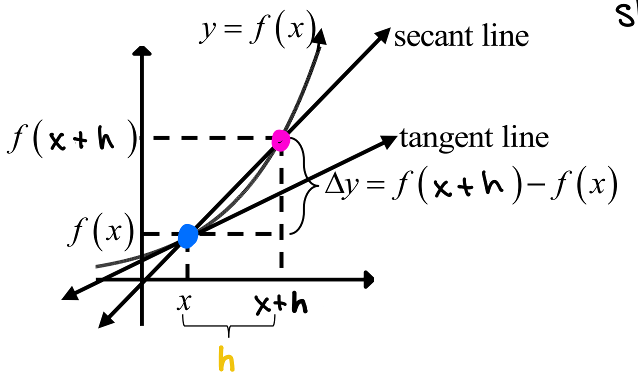
LESSON 2-4 LIMIT DEFINITION OF THE DERIVATIVE, ALTERNATE FORM OF THE LIMIT DEFINITION

Any nonvertical line has the same slope at every point. In Calculus we frequently deal with the slope of a curve. The slope of a curve is defined to be the same as the slope of the curve's tangent line at a given point. To find the slope of a tangent line we use a limit of the slope of a secant line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x}$$



$$m_{\sec} = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

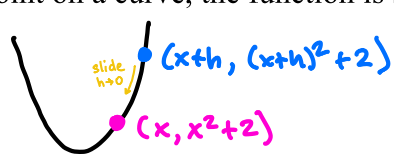
$$m_{\tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The slope of a tangent line is called the **<u>derivative</u>** of the function at a given x-value. The most commonly used symbol for the derivative is $f'(x)$. Here are some other notations you will encounter (assume $y = f(x)$).

$$\boxed{f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx} f(x) = m_{\tan} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

A <u>vertical tangent line</u> has no slope, so a curve has no derivative at any point where it has a vertical tangent line. <u>Differentiation</u> is the process of finding derivatives. If a derivative exists at a point on a curve, the function is said to be <u>differentiable</u> at that point.

<u>Examples:</u>



1. If $f(x) = x^2 + 2$

a. find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{[(x+h)^2 + 2] - [x^2 + 2]}{(x+h) - (x)}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+h) + 2 - x^2 - 2}{h}$$

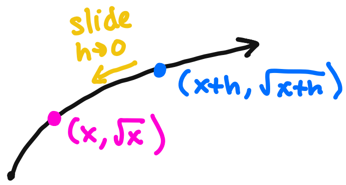
$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$ cross out the hole

$$= \lim_{h \to 0} 2x + h = 2x + 0 = 2x$$

b. use your answer from part a. to find $f'(-3)$.

$$f'(-3) = 2(-3) = -6$$



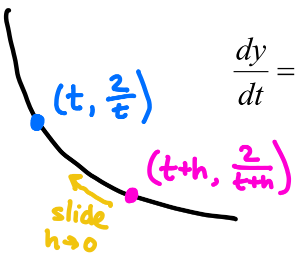
2. If $y=\sqrt{x}$, find y'.

$$y' = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{(x+h)-(x)}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+h}-\sqrt{x}\right)}{h} \frac{\left(\sqrt{x+h}+\sqrt{x}\right)}{\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h}+\sqrt{x}\right)}$$ cross out the hole $$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

3. Given $y=f(t)=\dfrac{2}{t}$, find the derivative of y <u>with respect to t</u>.



$$\frac{dy}{dt} = f'(t) = \lim_{h\to 0} \frac{\frac{2}{t+h}-\frac{2}{t}}{(t+h)-t}$$

$$= \lim_{h\to 0} \frac{\frac{(t)}{(t)}\frac{2}{t+h} - \frac{2}{t}\frac{(t+h)}{(t+h)}}{h}$$

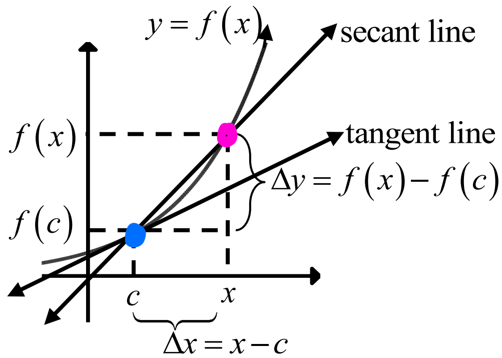
$$= \lim_{h\to 0} \frac{\frac{2t}{t(t+h)} - \frac{2(t+h)}{t(t+h)}}{h}$$

$$\lim_{h\to 0} \frac{2t-2t-2h}{t(t+h)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{t(t+h)\cancel{h}}$$ cross out hole

$$= \lim_{h\to 0} \frac{-2}{t(t+h)} = \frac{-2}{t(t+0)} = -\frac{2}{t^2}$$

<u>**Alternate Form of the Limit Definition of the Derivative**</u>
(Gives the value of the derivative at a <u>single</u> point.)

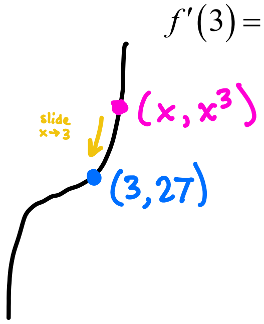


$$m_{\tan} = \lim_{x\to c} m_{\sec}$$

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

<u>Example 4.</u> If $f(x)=x^3$, use the alternate form of the derivative to find $\underbrace{f'(3)}$.

the slope at 3:00



$$f'(3) = \lim_{x\to 3} \frac{x^3-27}{x-3} = \lim_{x\to 3} x^2+3x+9$$

$$= 3^2+3(3)+9$$

$$= 27$$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 0 \end{array}$$