LESSON 5-6 DEFINITE INTEGRALS, CALCULATOR INTEGRATION, THE FUNDAMENTAL THEOREM OF CALCULUS

A Definite Integral is written with upper and lower limits attached to an integration expression such as $\int_a^b f(x)\,dx$.

The value of a definite integral $\left(\int_a^b f(x)\,dx\right)$ may be thought of as a "signed area" from the lower limit a (usually a left side boundary) to the upper limit b (usually a right-side boundary), and between the curve of $f(x)$ and the x-axis. The value may be positive, negative, or zero.
Unlike the previous integration process which produced an indefinite integral (an antiderivative) representing a family of curves, a definite integral represents **a number value**.

Calculator Integration: A TI-84 calculator can be used to find the value of a definite integral from a to b by using $\int f(x)\,dx$ in the calculate menu or fnInt in the math menu. The calculate menu shows a graphical representation of the "signed area" together with the value of the definite integral.

Examples:

Use the calculate menu to evaluate the following definite integrals.

1. $\int_{-3}^{1}(x^3-6x)\,dx = 4$ 2. $\int_{-\sqrt{6}}^{\sqrt{6}}(x^3-6x)\,dx = 0$ 3. $\int_{-5}^{5}\left|x^3-6x\right|dx = 198.501$

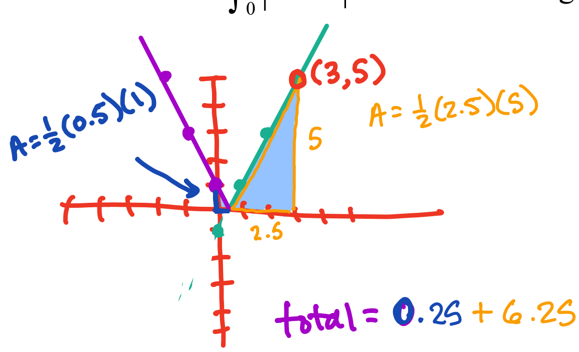
The math menu only provides the value of the definite integral, but that is usually all that we need. More importantly, the math menu gives a more accurate answer. **fnInt is recommended for all problems from now on**. **Note:** Newer operation systems have a MATHPRINT setting that simplifies this process.

Use the math menu to evaluate:

4. $\int_{-5}^{5}\left|x^3-6x\right|dx =$ fnInt$(\text{abs}(x^3-6x), x, -5, 5)$ or if $y_1 = \text{abs}(x^3-6x)$ is already entered on your calculator, fnInt$(y_1, x, -5, 5)$

198.500

5. Use the idea of "signed area" to evaluate $\int_0^3 |2x-1|\,dx$ without using a calculator.



total = 0.25 + 6.25 = 6.5

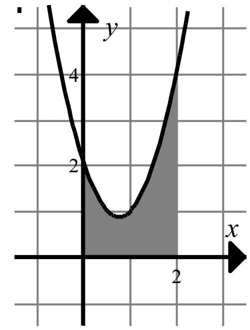
6. Set up a definite integral which could be used to find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$ (shown at right), the x-axis, and the vertical lines $x = 0$ and $x = 2$.

f

16

$\int_0^2 (2x^2-3x+2)\,dx$

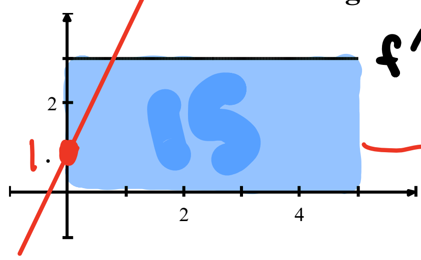
15



$y = 2x^2 - 3x + 2$

Discovering the Fundamental Theorem of Calculus

7.



a. Find $\int_0^5 f'(x)\,dx =$ 15

b. Write an equation for $f'(x)$ on $[0,5]$

$f'(x) = 3$

c. Find $f(x)$ if $f(1) = 4$.

$f(x) = 3x + C$

$f(1) = 3(1) + C = 4$

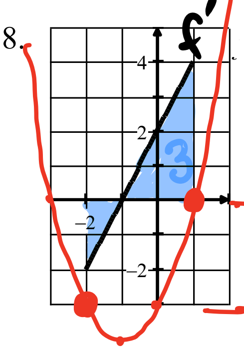
$C = 1$

$f(x) = 3x + 1$

d. Find $f(5) - f(0) =$

$16 - 1 = 15$

8.



a. Find $\int_{-2}^{1} f'(x)\,dx =$ 3

b. Write an equation for $f'(x)$ on $[-2,1]$. $f'(x) = 2x+2$

c. Find $f(x)$ if $f(1) = 0$.

$f(x) = x^2 + 2x + C$

$f(1) = (1)^2 + 2(1) + C = 0$

$C = -3$

$f(x) = x^2 + 2x - 3$

d. Find $f(1) - f(-2) =$

$-8 + 16 - 3$

$-4.5 + 12 - 3$

$0 - (-3) = 3$

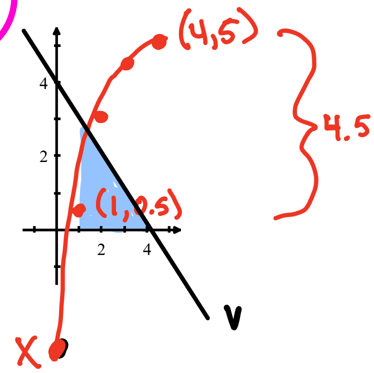
9. Given $x(t) = -\frac{1}{2}t^2 + 4t - 3$ is the position equation for an object moving on the x-axis.

a. Find the displacement of the object on the interval $[1,4]$ 4.5

b. Find the velocity equation $v(t) =$ $-t + 4$

c. Sketch a graph of $v(t)$

d. Find $\int_1^4 v(t)\,dt = \frac{1}{2}(3)(3) = 4.5$

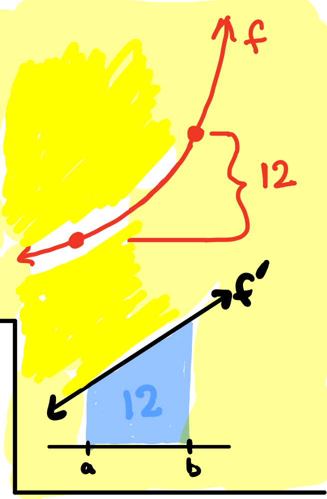


Notice for each of these, the answers to parts a and d are the same.

area vertical distance

If f' is a continuous function on $[a, b]$, then $\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b) - f(a)$

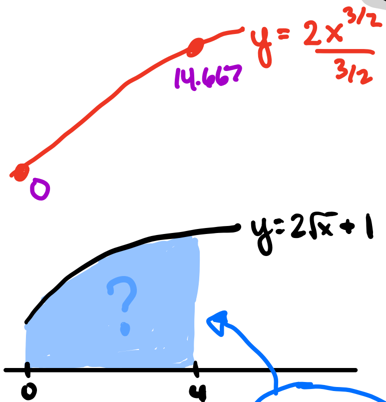
This relationship is known as the **Fundamental Theorem of Calculus**.



Evaluate using the Fundemental Theorem of Calculus without using a calculator.

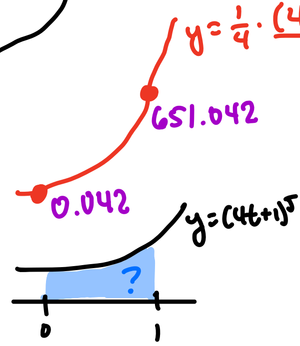
① can't find area of blue region :(
② integrate
③ plug in x=0 and x=4 to get vertical distance

10. $\int_0^4 \left(2\sqrt{x}+1\right)dx$



14.667 − 0 = 14.667 = 14.667

11. $\frac{1}{4}\int_0^1 (4t+1)^5\,dt \cdot 4$



651.042 − 0.042 = 651 = 651

12. $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$

$\int_1^5 x(2x-1)^{-1/2}\,dx$

$\int \frac{u+1}{2}(u)^{-1/2}\frac{du}{2}$

$\frac{1}{4}\int (u^{1/2} + u^{-1/2})\,du$

$u = 2x-1$
$x = \frac{u+1}{2}$
$\frac{du}{dx} = 2$
$du = 2\,dx$
$\frac{du}{2} = dx$

$\frac{1}{4}\left(\frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} + C\right)$

$\frac{1}{6}(2x-1)^{3/2} + \frac{1}{2}(2x-1)^{1/2} + C\Big|_1^5$

Plug in 5 − Plug in 1

6 − 2/3

= 16/3

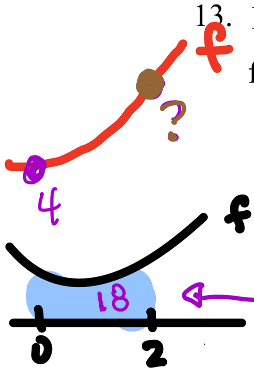
START PLUS ACCUMULATION METHOD

Since $\int_a^b f'(x)\,dx = f(b) - f(a)$, it follows that $\boxed{f(b) = f(a) + \int_a^b f'(x)\,dx}$.

This means a function value can be found as a starting value plus a definite integral.

Examples:

13. If $f'(x) = 3x^2 + 3$ and $f(0) = 4$, find $f(2)$ without a calculator.



$f(x) = x^3 + 3x + C\Big|_0^2$

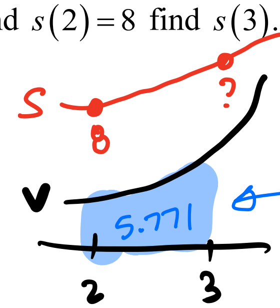
$= f(2) - f(0)$

$= (18 + C) - (0 + C)$

$= 18$

$f(2) = 4 + 18 = 22$

14. If an object's velocity is $v(t) = 2^t$ and $s(2) = 8$ find $s(3)$.



use calculator $\int_2^3 2^t\,dt$

$s(3) = 8 + 5.771$

$= 13.771$