name:

BC Topic 9 — Elementary Series

due Tuesday, November 28

- Ways to make power series:
  - rewrite them in geometric form (Topic 7)
  - use the Taylor formula (Topic 8)
  - just memorize them (Topic 9)

In this section you will be using the four elementary power series. You are expected to know them from memory.

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$

$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots + \frac{(-1)^{n-1}(x-1)^n}{n} + \cdots$

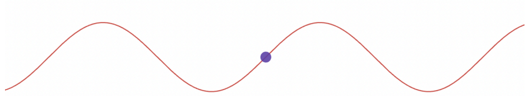
Creating new power series.

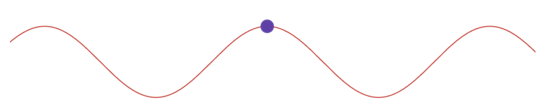
Examples: Using these elementary series, find a power series for each of the following functions. Show four terms and the general term.

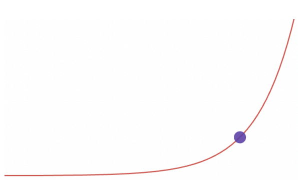
1. $\sin x^2 = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \frac{(x^2)^7}{7!} + \cdots + \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!} + \cdots$

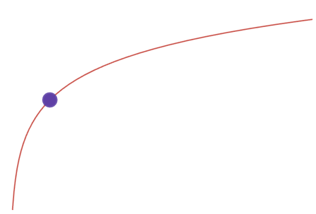
2. $\cos\sqrt{x} = 1 - \frac{(\sqrt{x})^2}{2!} + \frac{(\sqrt{x})^4}{4!} - \frac{(\sqrt{x})^6}{6!} + \cdots + \frac{(-1)^n (\sqrt{x})^{2n}}{(2n)!} + \cdots$

3. $xe^x = x + x^2 + \frac{x^3}{2} + \frac{x^4}{3!} + \cdots + \frac{x^{n+1}}{n!} + \cdots$









Use an elementary series to give a series for each of the following functions. Show four terms and the general term.

1. $f(x)=e^{-4x}$ 2. $g(x)=\cos(3x)$ 3. $f(x)=2\sin x^2$ 4. $h(x)=(x-1)\ln x$

Write each of the following series as a function using elementary functions.

5. $x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$

6. $1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\frac{x^8}{4!}-\cdots$

7. $\frac{x-1}{x}-\frac{(x-1)^2}{2x}+\frac{(x-1)^3}{3x}-\frac{(x-1)^4}{4x}+\cdots$

8. $1+x-\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}-\cdots$

Find the value of each of the following using an elementary function.

9. $1-\frac{\pi^2}{2!}+\frac{\pi^4}{4!}-\frac{\pi^6}{6!}+\cdots$

10. $(e-1)-\frac{(e-1)^2}{2}+\frac{(e-1)^3}{3}-\frac{(e-1)^4}{4}+\cdots$

11. $5+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\cdots$

12. $-\frac{2^3}{3!}+\frac{2^5}{5!}-\frac{2^7}{7!}+\frac{2^9}{9!}-\cdots$

Given $f(x)=1-\frac{4}{3}(x-2)^2+\frac{16}{5}(x-2)^4-\frac{2^6}{7}(x-2)^6+\cdots$ is a Taylor Series expansion for $f(x)$ find:

24. a general term for the series.

25. the center of the series.

26. $f(2)$

27. $f'(2)$

28. $f''(2)$

29. $f^{(11)}(2)$

30. $f^{(12)}(2)$

31. $f'(x)$

32. Is the point $(2,1)$ on this same function a local minimum, a local maximum, or neither. Justify.

38. Find a third degree Taylor Polynomial for $f(x)=\tan x$ centered at $c=\frac{\pi}{4}$.

39. Find a series for the function $f(x)=\cos x$ centered at $c=\frac{3\pi}{4}$. Show four terms.

40. Write the following function as a geometric series showing four terms and the general term and find the interval of convergence. $f(x)=\frac{3}{1+2x}$

41. Find a function for the series $g(x)=1-\frac{3}{4}x+\frac{9}{16}x^2-\frac{27}{64}x^3+\cdots$ and determine its domain.

1. $1-4x+8x^2-\frac{32}{3}x^3+\bullet\bullet\bullet+\frac{(-1)^n 4^n x^n}{n!}+\bullet\bullet\bullet$

2. $1-\frac{(3x)^2}{2!}+\frac{(3x)^4}{4!}-\frac{(3x)^6}{6!}+\bullet\bullet\bullet+\frac{(-1)^n 3^{2n} x^{2n}}{(2n)!}+\bullet\bullet\bullet$

4. $(x-1)^2-\frac{(x-1)^3}{2}+\frac{(x-1)^4}{3}-\frac{(x-1)^5}{4}+\bullet\bullet\bullet+\frac{(-1)^{n+1}(x-1)^{n+1}}{n}+\bullet\bullet\bullet$

5. $x\cos x$ 6. e^{-x^2} 7. $\frac{\ln x}{x}$ 9. -1 10. 1 11. e^4 12. -1.091

25. $c=2$ 27. 0 28. $-\frac{8}{3}$ 29. 0

31. $f'(x)=-\frac{8}{3}(x-2)+\frac{64}{5}(x-2)^3-\frac{2^6\bullet 6}{7}(x-2)^5+\cdots$

36. diverges 37. $\frac{3}{2}$ 38. $\tan x\approx 1+2\left(x-\frac{\pi}{4}\right)+\frac{4}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{16}{3!}\left(x-\frac{\pi}{4}\right)^3$

40. $f(x)=3-6x+12x^2-24x^3+\bullet\bullet\bullet+(-1)^n 3\bullet 2^n x^n+\bullet\bullet\bullet$, IOC: $-\frac{1}{2}<x<\frac{1}{2}$

41. $g(x)=\frac{1}{1+\frac{3}{4}x}$, Do: $-\frac{4}{3}<x<\frac{4}{3}$