BC Topic 5 — Direct Comparison Test
due Wednesday, October 18

← visualization of the Direct Comparison Test

Direct Comparison Test: Let $0 < a_n \le b_n$ for all n after a certain n.

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Informally:
1. If the "larger" series converges, then the "smaller" series must also converge.
2. If the "smaller" series diverges, then the "larger" series must also diverge.

Examples: Determine the convergence or divergence of the following.

7. $\sum_{n=1}^{\infty} \dfrac{1}{1+2^n}$

Compare to $\sum \dfrac{1}{2^n}$
GS $r = \frac{1}{2} < 1$ conv.
by GST

$\dfrac{1}{1+2^n} < \dfrac{1}{2^n}$

$\sum \dfrac{1}{1+2^n}$ conv.
by DCT

8. $\sum_{n=3}^{\infty} \dfrac{1}{n-2}$

Compare to $\sum \dfrac{1}{n}$
div. harm.

$\dfrac{1}{n-2} > \dfrac{1}{n}$

$\sum \dfrac{1}{n-2}$ div. by

DCT

9. $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n}+1} = \frac{1}{2} + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{5}+1} + \cdots$
$= .5 + .4\text{H} + .366 + .333 + .309 + \cdots$

Compare to $\sum \dfrac{1}{\sqrt{n}}$
$p = \frac{1}{2} < 1$ div. p-sT

$\dfrac{1}{\sqrt{n}+1} < \dfrac{1}{\sqrt{n}}$ (dead end)

Compare to $\sum \dfrac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$
$= 1 + .5 + .333 + .25 + .2$
div. harm.

$\dfrac{1}{\sqrt{n}+1} > \dfrac{1}{n}$, $n \ge 3$

$\sum \dfrac{1}{\sqrt{n}+1}$ div by DCT

Use the Direct Comparison Test to determine convergence or divergence.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4}$

2. $\displaystyle\sum_{n=5}^{\infty} \frac{1}{n-4}$

3. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{\sqrt{n}-1}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n+1}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{4^n+2}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n+2}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

Selected Answers:
1. conv.   2. div.   3. div.   4. conv.   5. conv.   6. div.   7. conv.