name:

BC Topic 4 — p-Series Test
due Thursday, October 5

p-Series and Harmonic Series:

If p is a positive constant then $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ is called a ***p*-series.**

The last three examples are all p-series. Each of them could have been done using the following test.

> ***p*-series Test**
> If $p > 1$ then the p-series **converges.** If $0 < p \le 1$ then the p-series **diverges.**

The **harmonic series** is the p-series in which $p = 1$. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$ (Example 3 above)

Examples: Use the p-series test to determine convergence or divergence of these series.

6. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$p = \frac{3}{2} > 1$ conv.
by p-s Test

7. $\sum_{n=1}^{\infty} n^3 \sqrt[3]{n^{-11}} = \sum_{n=1}^{\infty} n^3 \cdot n^{-\frac{11}{3}}$

$= \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$p = \frac{2}{3} < 1$
div. by p-s T

Use the p-series Test to show convergence or divergence.

9. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$

10. $1+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}+\dfrac{1}{4\sqrt{4}}+\cdots$

11. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^e}$

Determine the convergence or divergence by any method.

12. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^{\ln n}$

13. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[4]{n^5}}$

14. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5}{4}\right)^n$

15. $\displaystyle\sum_{n=1}^{\infty}\frac{2n}{\sqrt{n^2+2}}$

17. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^2}+\frac{1}{n^3}\right)$

18. $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{3^{n+1}}$

19. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[3]{n}}$

20. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(n+1)}{\sqrt[3]{n}}$

Selected Answers:

9. diverges by p-sT

10. converges by p-sT

12. diverges (harmonic series)

13. converges by p-sT

14. diverges by GST

18. converges by GST

19. converges by AST