name:

BC Topic 3 — Alternating Series Test
due Friday, September 22

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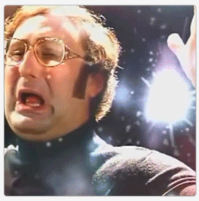

Converge or Explode 2

Alternating Series Test for Convergence:

Let $a_n > 0$. The alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converge if

1. $a_{n+1} \le a_n$ for all n after a certain n (terms never increase in absolute value) and
2. $\lim_{n\to\infty} a_n = 0$

Examples: Determine the convergence or divergence.

6. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$

terms dec. in abs. val.

$\lim_{n\to\infty}\frac{1}{n} = 0$

conv. by AST

7. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+1}{n} = 2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$

$\lim_{n\to\infty}\frac{n+1}{n} = 1 \neq 0$

div. by nTT

We now have three tests for Convergence/Divergence.

n^{th} Term Test for Divergence: If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

This test is inconclusive if $\lim_{n\to\infty} a_n = 0$. (cannot be used to show conv.)

Geom. Series Test: $|r| \ge 1 \rightarrow$ **diverges**, $|r| < 1 \rightarrow$ **converges** and $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.

Alternating Series Test for Convergence: converges if

1. $a_{n+1} \le a_n$ for all n after a certain n (terms never increase in absolute value)
2. $\lim_{n\to\infty} a_n = 0$

Note: AST connot be used for div.

Determine the convergence or divergence of each series. Show justification and name the test used. If possible, find the sum of the series.

13. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

14. $\sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{n}}{n^2+1}$

15. $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^3+2}$

16. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln n}$

17. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

18. $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

19. $\sum_{n=1}^{\infty} \frac{4n+2}{5n-1}$

20. $\sum_{n=1}^{\infty} 2\pi^{-n}$

21. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{5n^2-1}$

22. $\sum_{n=0}^{\infty} \frac{(-2)^n}{5^{n+1}}$

23. $1-\frac{\left(\frac{\pi}{4}\right)^2}{2!}+\frac{\left(\frac{\pi}{4}\right)^4}{4!}-\frac{\left(\frac{\pi}{4}\right)^6}{6!}+\cdots$

Write an expression for the nth term of these sequences. Assume $n = 1, 2, 3, \ldots$

33. $5, \frac{5}{2}, \frac{5}{6}, \frac{5}{24}, \frac{5}{120}, \cdots$

34. $\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \frac{1}{13}, \cdots$

Determine if the following sequences converge or diverge.

35. $\frac{5}{4}, \frac{8}{7}, \frac{11}{10}, \frac{14}{13}, \cdots$

36. $a_n = \frac{n^3}{n^2+2}$

37. Find the value of $2-\frac{2}{3}+\frac{2}{9}-\frac{2}{27}+\cdots$

13. converge by AST 14. converge by AST 15. diverge by nTT
17. converge by AST 18. converge by GST, Sum = 3 19. diverge by nTT
20. converge by GST, Sum = $\frac{2}{\pi-1}$ 21. converge by AST
22. converge by GST, Sum = $\frac{1}{7}$ 23. converge by AST, Sum = $\frac{1}{\sqrt{2}}$

33. $\frac{5}{n!}$ 35. converges to 1
36. diverges 37. $\frac{3}{2}$