name:

BC Topic 2 — Series
due Tuesday, September 12

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Converge or Explode?

Series:
Recall from the last lesson that a **sequence** is an **ordered list** of numbers. In this lesson we will work with a **series** which is a **sum** of numbers.

An infinite series can be represented as $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$

Example 1:

What happens as more and more terms of a series like $\sum_{n=0}^{\infty}(2n+1) = 1+3+5+7+\cdots$ are added?

The sum approaches infinity (diverges).

When this happens the series is called **divergent**.

Example 2:

What happens as more and more terms of $\sum_{n=1}^{\infty}\frac{3}{10^n} = \frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\ldots$ are added?

$= .333\ldots$

$= \frac{1}{3}$

This is an example of a **convergent** geometric series.

Geometric Series:
If consecutive terms in a series have a common ratio ***r***, the series is called a **geometric series**.

$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots + ar^n + \cdots$ is the general form of a geometric series.

first term!

If $|r| \geq 1$ the geometric series **diverges**. If $|r| < 1$ the geometric series **converges** and $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.

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Examples. Determine if these series converge or diverge and, if possible, find the sum of the series.

3. $\sum_{n=0}^{\infty} \frac{3}{2^n} = 3 + \frac{3}{2} + \frac{3}{4} + \cdots$

$r = \frac{1}{2}$ conv. by GST

$S = \frac{3}{1-\frac{1}{2}}$

$= 6$

4. $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

$r = \frac{3}{2}$

div. by GST

5. $\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n = -2 + 1 - \frac{1}{2} + \frac{1}{4}$

$r = -\frac{1}{2}$

$S = \frac{-2}{1-(-\frac{1}{2})}$

$= \frac{-2}{\frac{3}{2}} = -\frac{4}{3}$

Example 6. Find the fraction form of the repeating decimal $.\overline{08}$ using a geometric series.

$.\overline{08} = .08 + .0008 + .000008 + \cdots$

$= \frac{8}{100} + \frac{8}{10,000} + \frac{8}{1,000,000} + \cdots$

$a = \frac{8}{100}$ $\quad r = \frac{1}{100}$ $\quad .\overline{08} = \frac{\frac{8}{100}}{1-\frac{1}{100}} = \frac{8}{100} \cdot \frac{100}{99} = \frac{8}{99}$

In general, convergence of a series is less simple than convergence of a sequence,

The sequence $\{a_n\} = \left\{1 + \frac{1}{n}\right\}$ converges because $\lim_{n\to\infty} a_n = 1$

However the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right) = (1+1) + \left(1+\frac{1}{2}\right) + \left(1+\frac{1}{3}\right) + \cdots$ does **not** converge.

A series **cannot** converge unless the terms approach a limit of **zero**.

> **n^{th} Term Test for Divergence:**
>
> If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. This test is inconclusive if $\lim_{n\to\infty} a_n = 0$.

Example 7. Show that $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$ diverges.

$\lim_{n\to\infty} \frac{n!}{2n!+1} = \frac{1}{2} \neq 0$

diverges by nTT

Determine whether each of the following infinite series converges or diverges. Show justification and name the test being used. In addition find the sum of the series, if possible.

1. $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ 2. $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n$ 3. $\sum_{n=0}^{\infty} 5\left(\frac{3}{2}\right)^n$ 4. $\sum_{n=1}^{\infty} \frac{n^2+2}{n^2}$

5. $\sum_{n=2}^{\infty} \frac{n^2}{\ln n}$ 6. $5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\cdots$ 7. $3+\frac{9}{2}+\frac{27}{4}+\frac{81}{8}+\cdots$ 8. $1+0.2+0.04+0.008+\cdots$

9. $\sum_{n=1}^{\infty} \frac{n}{\sin n}$ 10. $\sum_{n=1}^{\infty} \frac{3^n+2}{3^{n+2}}$ 11. $\sum_{n=0}^{\infty} \frac{e^n}{\pi^{n+1}}$ 12. $\frac{1}{2}+\frac{2}{4}+\frac{6}{8}+\frac{24}{16}+\frac{120}{32}+\cdots$

13. $\sum_{n=1}^{\infty} \left(\sin e^{10}\right)^n$ 14. $\sum_{n=1}^{\infty} \frac{(-5)^n}{6}$ 15. $\sum_{n=1}^{\infty} \left(-\frac{5}{6}\right)^n$ 16. $18-12+8-\frac{16}{3}+\frac{32}{9}-\cdots$

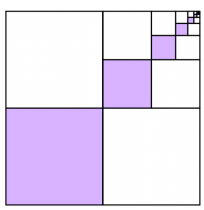
17. $\sum_{n=1}^{\infty} \frac{2n+3}{3n+2}$ 18. $\sum_{n=0}^{\infty} \frac{n!}{e^n}$ 19. $\sum_{n=0}^{\infty} \frac{4}{3^n}$ 20. $\sum_{n=1}^{\infty} 4^{-n}$

21. Given the series $\sum_{n=1}^{\infty} \frac{n^2+2}{n^3}$:

a. Find $\lim_{n\to\infty} a_n$.

b. Explain why the *nth* Term Test **cannot** be used to conclude the series converges.

22. This square is one inch wide. Find the area of the shaded region.



23. The first triangle is shaded 25%. What percent of the triangle will be shaded after infinitely many steps?

Determine the convergence/divergence of the following sequences:

24. $\left\{\left(1+\frac{2}{n}\right)^n\right\}$ 25. $\left\{\frac{(n+1)!}{n!}\right\}$

Write an expression for the nth term of the following sequences: (assume $n = 1, 2, 3, \cdots$)

26. $3, 7, 11, 15, \cdots$ 27. $1, \frac{-1}{2}, \frac{1}{6}, \frac{-1}{24}, \frac{1}{120}, \cdots$ 28. $2, 6, 18, 54, \cdots$

1. converges by GST, Sum = 15 2. conv. by GST, Sum = 10 3. div. by GST or nTT
5. div. by nTT 6. conv. by GST, Sum = 10 7. div. by GST 8. conv. by GST, Sum = 1.25
9. div. by nTT 11. conv. by GST, Sum = $\frac{1}{\pi - e}$ 13. conv. by GST, Sum = -.407 or -.408
14. div. by GST or nTT 15. conv. by GST, Sum = $-\frac{5}{11}$ 17. div. by nTT
18. div. by nTT 20. conv. by GST, Sum = $\frac{1}{3}$
24. converges to a limit of e^2 25. diverges 26. $a_n = 4n-1,\ n = 1, 2, 3, \cdots$
28. $a_n = 2(3)^{n-1},\ n = 1, 2, 3, \cdots$