BC Topic 20 - Partial Fractions
due Monday, May 6

We have often simplified an expression like $\dfrac{1}{x-4} - \dfrac{1}{x-3}$ by getting a common denominator and combining the two fractions into one. By a reverse process we can sometimes split a single fraction in two to make integration easier.

Example 1:

$$\int \frac{1}{x^2 - 7x + 12} dx$$

$(x-3)(x-4)$

$\dfrac{A(x-3)(x-4)}{x-3} + \dfrac{B(x-3)(x-4)}{x-4} = \dfrac{1}{x^2-7x+12} (x-3)(x-4)$

$A(x-4) + B(x-3) = 1$

let $x = 4 \to A\cdot 0 + B\cdot 1 = 1 \to B = 1$

let $x = 3 \to A(-1) + B\cdot 0 = 1 \to A = -1$

$\int \left(\dfrac{-1}{x-3} + \dfrac{1}{x-4} \right) dx$

$-\ln|x-3| + \ln|x-4| + C$

Example 2:

$$\int \frac{5x-3}{x^2 - 2x - 3} dx$$

$(x-3)(x+1)$

$\dfrac{A}{x-3} + \dfrac{B}{x+1} = \dfrac{5x-3}{(x-3)(x+1)}$

$A(x+1) + B(x-3) = 5x - 3$

$x = -1 \to -4B = -8$
$B = 2$

$x = 3 \to 4A = 12$
$A = 3$

$\int \left(\dfrac{3}{x-3} + \dfrac{2}{x+1} \right) dx$

$3\ln|x-3| + 2\ln|x+1| + C$

Example 3:

$$\int \frac{2x-2}{x^2 - 2x - 3} dx$$

$\ln|x^2 - 2x - 3| + C$

Example 4:

$$\int \frac{x^3 - x + 2}{x^2 + x - 2} dx$$

$x - 1 + \dfrac{2x}{x^2 + x - 2}$

$x^2 + x - 2 \overline{)x^3 \qquad -x + 2}$

$\underline{x^3 + x^2 - 2x}$

$-x^2 + x + 2$

$\underline{-x^3 - x + 2}$

$2x$

$\dfrac{A}{x+2} + \dfrac{B}{x-1} = \dfrac{2x}{x^2 + x - 2}$

$A(x-1) + B(x+2) = 2x$

$x = 1 \to 3B = 2 \to B = \frac{2}{3}$

$x = -2 \to -3A = -4 \to A = \frac{4}{3}$

$\int \left(x - 1 + \dfrac{\frac{4}{3}}{x+2} + \dfrac{\frac{2}{3}}{x-1} \right) dx$

$\frac{1}{2}x^2 - x + \frac{4}{3}\ln|x+2| + \frac{2}{3}\ln|x-1| + C$

Integrate without using a calculator.

1. $\int \dfrac{1}{x^2-1}\,dx$

2. $\int \dfrac{3}{x^2-x-2}\,dx$

3. $\int \dfrac{5x-2}{2x^2-x-1}\,dx$

4. $\int \dfrac{2x^2+2x-2}{x^3-x}\,dx$

5. $\int 3x\ln x\,dx$

6. $\int x^2\sin(3x)\,dx$

7. $\int \dfrac{2x-\sqrt{x}+3}{\sqrt{x}}\,dx$

8. $\int (2x+1)^6\,dx$

9. $\int (3t^2-1)^2\,dt$

10. $\int \dfrac{\sqrt{\ln y}}{y}\,dy$

11. $\int \dfrac{\sec^2\theta}{1+\tan\theta}\,d\theta$

12. $\int \dfrac{\sec^2\theta}{1+\tan^2\theta}\,d\theta$

---

1. $\dfrac{1}{2}\ln|x-1|-\dfrac{1}{2}\ln|x+1|+C$

2. $\ln\left|\dfrac{x-2}{x+1}\right|+C$

3. $\dfrac{3}{2}\ln|2x+1|+\ln|x-1|+C$

4. $2\ln|x|+\ln|x-1|-\ln|x+1|+C$

5. $\dfrac{3}{2}x^2\ln x-\dfrac{3}{4}x^2+C$

6. $-\dfrac{1}{3}x^2\cos(3x)+\dfrac{2}{9}x\sin(3x)+\dfrac{2}{27}\cos(3x)+C$

7. $\dfrac{4}{3}x^{\frac{3}{2}}-x+6x^{\frac{1}{2}}+C$

8. $\dfrac{1}{14}(2x+1)^7+C$

9. $\dfrac{9}{5}t^5-2t^3+t+C$

10. $\dfrac{2}{3}(\ln y)^{\frac{3}{2}}+C$

12. $\theta+C$