name:

BC Topic 1 — Sequences

due Wednesday, August 30

Factorial Definition: $n! = n(n-1)(n-2)(n-3)\cdots 1$

Examples;

1. Simplify $\frac{(n+2)!}{n!} =$ $\frac{(n+2)(n+1)\cancel{(n)}\cancel{(n-1)}\cancel{\cdots}}{\cancel{n}\cancel{(n-1)}\cancel{\cdots}}$

$= (n+2)(n+1)$

A **sequence** is an ordered list of numbers.

Examples:

2. Write out the first five terms of the sequence $\{a_n\}$ if $a_n = \frac{n}{n+1}$.

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

3. Write out the first five terms of the recursively defined sequence.

$a_1 = 5, \quad a_{k+1} = 3a_k + 4$

$5, 19, 61, 187, 565$

4. Write a recursive definition for the sequence $2, -6, 18, -54, 162, \cdots$

$a_1 = 2, \quad a_{k+1} = -3a_k$

5. Write an expression for the nth term of the sequence .

$a_n = 2(-3)^{n-1}$

Convergence or Divergence of a Sequence

If $\{a_n\}$ is a sequence and $\lim_{n\to\infty} a_n = L$ then L is the limit of the sequence and it **converges** to L.

Example:

6. Find the limit of the sequence $\{b_n\} = \left\{\frac{n}{1-2n}\right\}$.

$\lim_{n\to\infty} \frac{n}{1-2n} = -\frac{1}{2}$

If $\lim_{x\to\infty} a_n$ does not exist, then the sequence $\{a_n\}$ does not have a limit and $\{a_n\}$ **diverges**.

Examples: Determine if these sequences converge or diverge and find the limit if possible.

7. $\{a_n\} = \{3 + (-1)^n\}$

$\{a_n\} = \{2, 4, 2, 4, \ldots$

diverges

8. $\{a_n\} = \left\{\frac{\ln(n^2)}{n}\right\}$

$\lim_{n\to\infty} \frac{\ln(n^2)}{n}$

$\lim_{n\to\infty} \frac{2\ln n}{n} = 0$ converges

Without using a calculator, write the first five terms of the sequence with the given *nth* term. Assume $n = 1,2,3,\cdots$

1. $a_n = \dfrac{2^n}{n!}$ 2. $a_n = \left(-\frac{1}{3}\right)^n$ 3. $a_n = \cos\left(\dfrac{n\pi}{2}\right)$

Write the first five terms of the recursive sequence.

4. $a_1 = 2,\ a_{n+1} = 3(a_n + 2)$ 5. $a_1 = 0,\ a_{n+1} = \frac{\pi}{2}\left(\sin\left(a_n + \frac{\pi}{2}\right)\right)$

Write a recursive definition of the sequence.

6. $4, 7, 10, 13, \cdots$ 7. $5, 10, 20, 40, \cdots$ 8. $5, -\frac{5}{2}, \frac{5}{4}, -\frac{5}{8}, \cdots$

Simplify without using a calculator.

9. $\dfrac{7!}{10!}$ 10. $\dfrac{(2n+1)!}{(2n-1)!}$

Find the limit of each sequence or state that the sequence diverges.

11. $a_n = \dfrac{n^2}{3n^2 - 5}$ 12. $a_n = \dfrac{\ln n^2}{3n}$ 13. $a_n = \cos\dfrac{1}{n}$ 14. $a_n = (-1)^n \dfrac{n^2}{n^2+2}$

15. $a_n = \dfrac{3n}{\sqrt{n^2 - 5}}$ 16. $a_n = \dfrac{\cos n}{n}$ 17. $a_n = (-1)^n \dfrac{n}{n^2+2}$ 18. $a_n = \dfrac{(n+1)!}{n!}$

Write an expression for the *nth* term of each sequence. Assume $n = 1,2,3,\cdots$

19. $-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, \cdots$ 20. $\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \cdots$ 21. $\frac{2}{1}, \frac{4}{3}, \frac{8}{7}, \frac{16}{15}, \cdots$ 22. $\frac{3}{1}, \frac{3}{2}, \frac{3}{6}, \frac{3}{24}, \cdots$

23. $\frac{1}{2}, \frac{x}{6}, \frac{x^2}{24}, \frac{x^3}{120}, \cdots$ 24. $-1, 1, 3, 5, \cdots$ 25. $\frac{1}{1}, \frac{4}{3}, \frac{9}{9}, \frac{16}{27}, \cdots$

Selected Answers:

1. $2, 2, \frac{4}{3}, \frac{2}{3}, \frac{4}{15}$ 2. $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, -\frac{1}{243}$ 4. 2, 12, 42, 132, 402

6. $a_1 = 4,\ a_{n+1} = a_n + 3$ 9. $\frac{1}{720}$ 11. $\frac{1}{3}$ 13. 1 14. The sequence diverges.

16. 0 17. 0 19. $a_n = \dfrac{(-1)^n}{n^2}$ 22. $a_n = \dfrac{3}{n!}$ 25. $a_n = \dfrac{n^2}{3^{n-1}}$