name:

BC Topic 19 - Integration by Parts
due Wednesday, April 24

Integration by parts is a method of integration used mainly for products of algebraic and transcendental functions (such as $\int xe^x\,dx$) or products of two transcendental functions (such as $\int e^x \sin x\,dx$).

Development of the formula for integration by parts: If u and v are both functions of x, then

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$\int \frac{d}{dx}(uv)\,dx = \int u\frac{dv}{dx}\,dx + \int v\frac{du}{dx}\,dx$

$uv = \int u\,dv + \int v\,du$

$uv - \int v\,du = \int u\,dv$

Formula for integration by parts:
$\int uv'\,dx = uv - \int vu'\,dx$ or $\int u\,dv = uv - \int v\,du$

remember "ultra violet voodoo"

Strategy: Let u be the part whose derivative is "simpler" (or at least no more complicated) than u itself. Let dv be the more complicated part (or the part which can easily be integrated). Also, remember that you typically have only two choices. If one choice doesn't work, try the other.

Example 1. $\int xe^x\,dx$

Let $u = x$
$du = dx$
Let $dv = e^x dx$
$v = e^x$

$uv - \int v\,du$

$\int xe^x\,dx = xe^x - \int e^x dx$

$= xe^x - e^x$

Example 2. $\int x\sin(3x)\,dx$

let $u = x$
$du = dx$
let $dv = \sin(3x)$
$v = -\frac{1}{3}\cos(3x)$

$\int x\sin(3x)\,dx = x(-\frac{1}{3}\cos(3x)) - \int -\frac{1}{3}\cos(3x)\,dx$

$= -\frac{1}{3}x\cos(3x) + \frac{1}{3}\int \cos(3x)\,dx$

$= -\frac{1}{3}x\cos(3x) + \frac{1}{3}\cdot\frac{1}{3}\sin(3x) + C$

Example 3. $\int \arcsin x \, dx =$

let $u = \arcsin x$
$du = \frac{1}{\sqrt{1-x^2}} dx$
$dv = dx$
$v = x$

$\int \arcsin x \, dx = x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} dx$
$= x \arcsin x - \left(-\frac{1}{2}\right) \int -2x (1-x^2)^{-\frac{1}{2}} dx$
$= x \arcsin x + \frac{1}{2} \cdot \frac{2}{1} (1-x^2)^{\frac{1}{2}} + C$

Example 4. $\int x^2 \sin(2x) dx =$

$u_1 = x^2$
$du_1 = 2x dx$
$dv_1 = \sin(2x)$
$v_1 = -\frac{1}{2}\cos(2x)$

$x^2\left(-\frac{1}{2}\cos(2x)\right) - \int -\frac{1}{2}\cos(2x) 2x dx$
$= -\frac{1}{2}x^2 \cos(2x) + \int x \cos(2x) dx$

$u_2 = x$ $dv_2 = \cos(2x)$
$du_2 = dx$ $v_2 = \frac{1}{2}\sin(2x)$

$= -\frac{1}{2}x^2 \cos(2x) + x \cdot \frac{1}{2}\sin(2x) - \int \frac{1}{2}\sin(2x) dx$
$= -\frac{1}{2}\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$

Example 5. $\int_1^e x^2 \ln x \, dx$

$u = \ln x$ $dv = x^2 dx$
$du = \frac{1}{x} dx$ $v = \frac{1}{3}x^3$

$\left(\ln x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx\right)\Big|_1^e$
$\left(\frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx\right)\Big|_1^e$
$\left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right)\Big|_1^e = \left(\frac{1}{3}e^3 \ln e - \frac{1}{9}e^3\right) - \left(\frac{1}{3}\ln 1 - \frac{1}{9}\right)$
$= \frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9}$

Example 6. Complete the square to find

$$\int \frac{1}{x^2+4x+8} dx$$

$$\int \frac{1}{x^2+4x+4+4} dx$$

$$\int \frac{1}{(x+2)^2+4} dx$$

$$\frac{1}{2}\tan^{-1}\frac{x+2}{2} + C$$

Integrate without using a calculator. Integration by parts will be used on most, but **not all** problems.

1. $\int x\sin x\,dx$
2. $\int x\cos(2x)dx$
3. $\int 4xe^{2x}\,dx$
4. $\int x^2e^{x^3}\,dx$
5. $\int \frac{x}{e^x}dx$
6. $\int \ln x\,dx$
7. $\int \frac{\ln x}{x^2}dx$
8. $\int \frac{(\ln x)^2}{x}dx$
9. $\int \arctan x\,dx$
10. $\int x^2\cos x\,dx$
11. $\int x^2e^{2x}\,dx$
12. $\int_0^1 \frac{x}{1+x^2}dx$
13. $\int_{-3}^{-1} \frac{1}{\sqrt{7-x^2-6x}}dx$
14. $\int_0^1 xe^{3x}\,dx$
15. $\int_1^{e^2} x\ln x\,dx$
16. $\int x\arctan x\,dx$ *

*$\int \frac{x^2}{1+x^2}dx$ can be integrated using long division

1. $-x\cos x+\sin x+C$
2. $\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)+C$
3. $2xe^{2x}-e^{2x}+C$
5. $-xe^{-x}-e^{-x}+C$
6. $x\ln x-x+C$
7. $-\frac{1}{x}\ln x-\frac{1}{x}+C$
8. $\frac{1}{3}(\ln x)^3+C$
9. $x\arctan x-\frac{1}{2}\ln(1+x^2)+C$
10. $x^2\sin x+2x\cos x-2\sin x+C$
11. $\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C$
12. $\frac{1}{2}\ln 2$
13. $\frac{\pi}{6}$
14. $\frac{2}{9}e^3+\frac{1}{9}$
15. $\frac{3}{4}e^4+\frac{1}{4}$
16. $\frac{1}{2}x^2\arctan x-\frac{1}{2}x+\frac{1}{2}\arctan x+C$