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BC Topic 17 - Logistic Equations
due Tuesday, March 26

Exponential growth modeled by $y = Ce^{kt}$ assumes unlimited growth and is unrealistic for most population growth. More typically the growth rate decreases as the population grows and there is a maximum population M called the carrying capacity. This is modeled by the **logistic** differential equation $\frac{dP}{dt} = kP(M-P)$.

The solution equation is of the form $P = \frac{M}{1+Ce^{-kMt}}$. **Note:** Unlike in the exponential growth equation, C is **not** the initial amount.

Example: A national park is capable of supporting no more than 100 grizzly bears. We model the equation with a logistic differential equation with $k = 0.001$.

a. Write the differential equation.

$\frac{dP}{dt} = .001P(100-P)$

b. The slope field for this differential equation is shown. Where does there appear to be a horizontal asymptote?

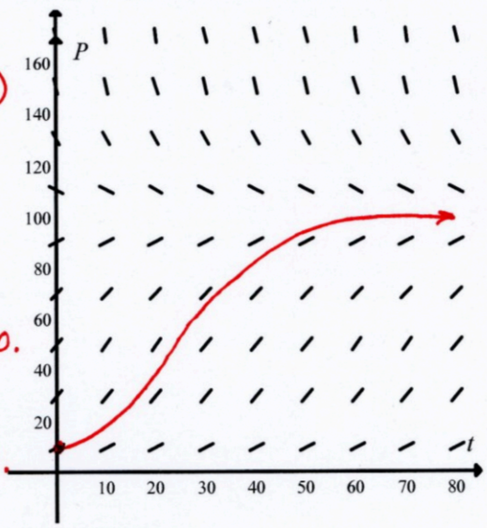
$P = 100$

What happens if the starting point is above this asymptote?

The population decreases toward 100.

What happens if the starting point is below this asymptote?

The population increases toward 100.



c. If the park begins with ten bears, sketch a graph of $P(t)$ on the slope field.

d. Write an equation to match the graph of *P(t)*.

$P = \frac{100}{1+Ce^{-(.001)(100)t}}$

$P = \frac{100}{1+Ce^{-.1t}}$

$P(0) = 10$

$10 = \frac{100}{1+C}$

$C = 9$

$P = \frac{100}{1+9e^{-.1t}}$

e. Find $\lim_{t\to\infty} P(t)$

$\lim_{t\to\infty} \frac{100}{1+9e^{-.1t}}$

$\frac{100}{1+0}$

100

f. When will the bear population reach 50?

$50 = \frac{100}{1+9e^{-.1t}}$

$t = 21.972$ yrs.

g. When is the bear population growing the fastest.

when $P = 50$

$t = 21.972$ yrs.

Assignment

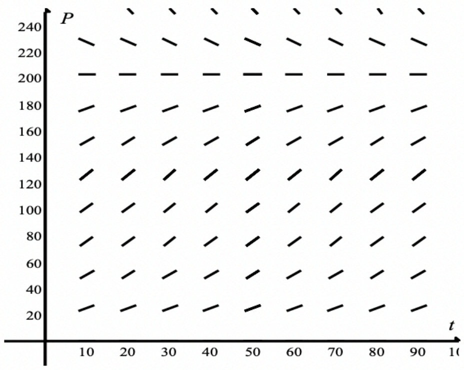
1. Logistic growth has $k = 0.00025$, $M = 200$, and $P(0) = 10$.

   a. Write a differential equation for the population.

   b. Sketch the population function on the slope field.

   c. Find a formula for P.

   d. When is the population growing the fastest?



2. If $\frac{dP}{dt} = .04P - .0004P^2$ find k and the carrying capacity.

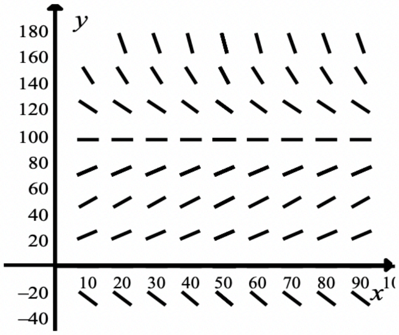
Match each of these differential equations with one of slope fields shown.

3. $\frac{dy}{dx} = .065y$

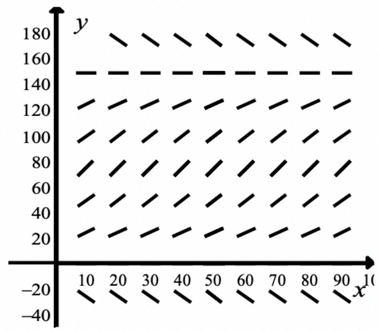
4. $\frac{dy}{dx} = .0006y(100 - y)$

5. $\frac{dy}{dx} = .06y\left(1 - \frac{y}{150}\right)$

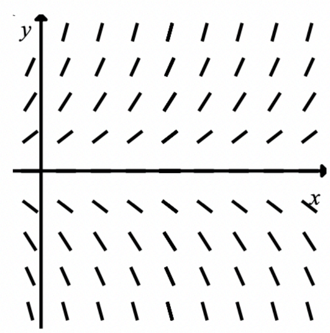
A.



B.



C.



6. Given the logistic equation $P(t) = \frac{2000}{1 + 19e^{-0.6t}}$:

   a. find the carrying capacity.

   b. find the value of k.

   c. find the initial population.

   d. find the time at which the population reaches 500.

   e. give the logistic differential equation.

7. Given the logistic differential equation $\frac{dP}{dt} = .03P(100 - P)$:

   a. find the value of k.

   b find the carrying capacity.

   c. find the value of P when $\frac{dP}{dt}$ is the greatest.

8. Given the logistic differential equation $\frac{dy}{dt} = 2y\left(1 - \frac{y}{50}\right)$:

   a. find the value of k.

   b. find the value of M.

   c. give the logistic equation if $y(0) = 10$.

9. A 200 gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is $\frac{dP}{dt} = .0015P(150 - P)$, where time t is measured in weeks.

   a. Find a formula for the guppy population in terms of t.

   b. How long will it take for the guppy population to be 100? 125?

10. The amount of food placed daily into a biology lab enclosure can support no more than 200 fruit flies. A biologist releases 25 flies into the enclosure. Four days later she counts 94 flies.
   a. Give the logistic equation.
   b. Find the number of flies on the 7th day.
   c. When will there be 175 flies?
   d. Find the logistic differential equation.
   e. Find the population and the time at which the growth is the fastest.

1a. $\frac{dP}{dt} = .00025P(200 - P)$ c. $P(t) = \frac{200}{1 + 19e^{-.05t}}$ 2. $k = .0004$, $M = 100$

6a. 2000 b. .0003 c. 100 d. 3.076 e. $\frac{dP}{dt} = .0003P(2000 - P)$ 8c. $y = \frac{50}{1 + 4e^{-2t}}$

9a. $P(t) = \frac{150}{1 + 24e^{-.225t}}$ 10a. $P(t) = \frac{200}{1 + 7e^{-.456t}}$ b. 155 flies c. 8.526 days

10d. $\frac{dP}{dt} = .002P(200 - P)$