name:

BC Topic 16 - Improper Integrals
due Thursday, March 14

An integral is called **improper** if
1. one or both limits of integration are infinite **(infinitely wide)**
2. the function has an infinite discontinuity (a vertical asymptote) at or between the limits **(infinitely tall)**

Explain why each of the following is improper.

1. $\int_1^\infty \frac{1}{x}\,dx$ 2. $\int_{-\infty}^\infty \frac{1}{x^2+1}\,dx$ 3. $\int_1^5 \frac{1}{\sqrt{x-1}}\,dx$ 4. $\int_{-2}^2 \frac{1}{(x+1)^2}\,dx$

infinite   infinite   undef. (VA)   undef. (VA)
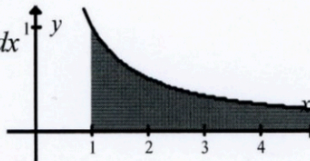limit   limits   at limit   at x = -1

Evaluate the following improper integrals. Identify those which diverge.

1. $\int_1^\infty \frac{1}{x^2}\,dx$ 2. $\int_1^\infty \frac{1}{x}\,dx$ 3. $\int_0^\infty \cos x\,dx$
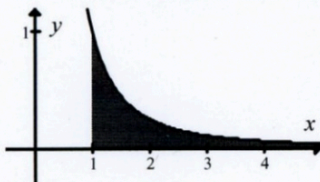
$\lim\limits_{b\to\infty} \int_1^b x^{-2}\,dx$

$\lim\limits_{b\to\infty} (-x^{-1})\Big|_1^b$

$\lim\limits_{b\to\infty} \frac{-1}{b} + 1$

1

$\lim\limits_{b\to\infty} \int_1^b \frac{1}{x}\,dx$

$\lim\limits_{b\to\infty} \ln|x|\Big|_1^b$

$\lim\limits_{b\to\infty} \ln b - \ln 1$

diverges

$\lim\limits_{b\to\infty} \int_0^b \cos x\,dx$

$\lim\limits_{b\to\infty} \sin x\Big|_0^b$

$\lim\limits_{b\to\infty} \sin b - \sin 0$

diverges

5. $\int_{-1}^2 \frac{dx}{x^3} = \int_{-1}^0 x^{-3}\,dx + \int_0^2 x^{-3}\,dx$

$\lim\limits_{b\to0^-} \int_{-1}^b x^{-3}\,dx + \lim\limits_{a\to0^+} \int_a^2 x^{-3}\,dx$

$\lim\limits_{b\to0^-} \left(-\tfrac{1}{2}x^{-2}\right)\Big|_{-1}^b + \lim\limits_{a\to0^+} \left(-\tfrac{1}{2}x^{-2}\right)\Big|_a^2$

$\lim\limits_{b\to0^-} \frac{-1}{2b^2} + \tfrac{1}{2} + \lim\limits_{a\to0^+} \left(-\tfrac{1}{8} + \tfrac{1}{2a^2}\right)$

diverges

Assignment

Which of the integrals in Problems 1-4 is/are improper. For any which are improper identify why the integral is improper. **Do not evaluate the integrals.**

1. $\displaystyle\int_0^9 \frac{x+3}{\sqrt[3]{x}}\,dx$
2. $\displaystyle\int_0^1 \frac{x^3}{3x-2}\,dx$
3. $\displaystyle\int_0^4 \frac{1}{x^2-4x-5}\,dx$
4. $\displaystyle\int_0^\infty \frac{x}{e^x}\,dx$

Evaluate these improper integrals or show that the integral diverges without using a calculator. Show correct limit symbolism.

5. $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}\,dx$
6. $\displaystyle\int_2^{11} \frac{1}{\sqrt{(x-2)^3}}\,dx$
7. $\displaystyle\int_0^\infty \frac{4}{e^x}\,dx$
8. $\displaystyle\int_{-\infty}^0 e^{3x}\,dx$

9. $\displaystyle\int_1^\infty \frac{3}{x^4}\,dx$
10. $\displaystyle\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$

13. $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^4}\,dx$
14. $\displaystyle\int_0^\infty \frac{e^x}{e^x+3}\,dx$

18. $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$
19. $\displaystyle\int_0^4 \frac{1}{(x-1)^2}\,dx$

20. A region R satisfies the inequalities $y \le \dfrac{1}{x^2}$, $y \ge 0$, $x \ge 1$.

    a. Sketch the region.
    b. Find the area of the region if possible.
    c. Find the volume of the solid formed by revolving the region about the x-axis if possible.
    d. Find the volume of the solid formed by revolving the region about the y-axis if possible.

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1. improper (V.A. at $x=0$)   2. improper (V.A. at $x=\frac{2}{3}$)   3. not improper

4. improper (infinite limit)   5. 6   6. diverges   7. 4   8. $\frac{1}{3}$ 9. 1

                          13. $\frac{1}{3}$ 14. diverges   18. $\frac{9}{2}$

19. diverges