name:

BC Topic 15 - Vector Calculus
due Friday, March 1

Position vector- $\langle x(t), y(t) \rangle$ gives the location of the object

Speed- $\|v(t)\| = \sqrt{(x'(t))^2 + (y'(t))^2}$

Velocity vector- $\langle x'(t), y'(t) \rangle$

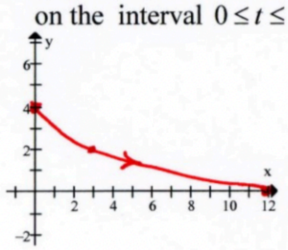
Distance traveled- $\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

Acceleration vector- $\langle x''(t), y''(t) \rangle$

"at rest" $v(t) = \langle 0, 0 \rangle$

Example 1. Given a position vector $\langle 3t^2, t^3 - 3t^2 + 4 \rangle$ for a particle moving in the xy-plane find the following.

a. graph the path of the particle on the interval $0 \le t \le 2$



b. the velocity vector at time $t = 1$

$v(t) = \langle 6t, 3t^2 - 6t \rangle$

$v(1) = \langle 6, -3 \rangle$

c. the speed of the particle at time $t = 1$

$\text{Speed}(1) = \sqrt{6^2 + (-3)^2}$
$= \sqrt{45}$

d. the distance traveled between $t = 0$ and $t = 3$

$T.D. = \int_0^3 \sqrt{(6t)^2 + (3t^2 - 6t)^2}\, dt$
$= 28.583$

e. the time(s) when the particle is at rest

$6t = 0$ and $3t^2 - 6t = 0$
$t = 0$ and $t = 0, 2$

$\boxed{t = 0}$

f. the acceleration vector at time $t = 2$

$a(t) = \langle 6, 6t - 6 \rangle$

$a(2) = \langle 6, 6 \rangle$

g. the direction of the particle at time $t = 1$ and when $t = 2$

$s(1) = \langle 3, 2 \rangle \qquad \theta = \tan^{-1} \frac{2}{3}$
$= .588$

$s(2) = \langle 12, 0 \rangle \qquad \theta = \tan^{-1} \frac{0}{12}$
$= 0$

Assignment

1. The position of a particle in the *xy*-plane is given by $x = 4t^2$ and $y = \sqrt{t}$. Find the following:

   a. the velocity vector at $t = 4$
   b. the acceleration vector at $t = 4$
   c. the speed of the particle at $t = 4$
   d. the distance the particle moves between $t = 0$ and $t = 4$
   e. the direction of the particle at $t = 4$

2. The position of a particle is given by $x = t^2$ and $y = t^3$. Find the following:

   a. the speed of the particle at $t = 2$
   b. the direction of the particle at $t = 2$
   c. the distance the particle moves between $t = 1$ and $t = 4$
   d. the velocity vector at $t = 3$
   e. the acceleration vector at $t = 4$

3. A particle moves in the *xy*-plane so that at time t its velocity vector is $v(t) = \langle t^3, \cos(\pi t) \rangle$ and the particle's position vector at time $t = 0$ is $\langle 2,1 \rangle$.

   a. What is the position vector of the particle when $t = 2$?
   b. What is the acceleration vector of the particle when $t = 2$?
   c. What is the direction of the particle when $t = 1$?
   d. What is the distance the particle travels between $t = 0$ and $t = 2$?
   e. When is the particle at rest?
   f. What is the speed of the particle when $t = 2$?

4. A particle moves on the *xy*-plane so that at time t its coordinates are $x = t^3 + t$ and $y = t^5 - 2t^2$. Find its velocity vector at time $t = 2$.

5. A calculator is allowed on this problem.
   The position of an object moving on a curve is $(x(t), y(t))$ at time t.

   Given $\frac{dx}{dt} = \sqrt{\frac{t}{2+t}}$ and $\frac{dy}{dt} = \cos(t^2 - 1)$. At time $t = 1$, the position of the object is $(2,4)$.

   a. Find the position of the object at time $t = 3$.
   b. Find the speed of the object at time $t = 3$.
   c. Find the total distance traveled by the object over the interval $1 \le t \le 3$.
   d. Find an equation of the line tangent to the curve at time $t = 3$.
   e. Find the acceleration vector at time $t = 3$.

6. Given a parametric curve defined by $x = e^t$ and $y = t + 1$.
   a. Find the length of the arc of the curve on the interval $1 \le t \le 6$.
   b. Find an equation of the line tangent to the curve when $t = 1$.
   c. Is the curve concave upward or downward when $t = 2$?
   d. Give a rectangular equation for the curve .

7. Find all points of horizontal and vertical tangency to the curve $x = 2 - 2\cos\theta,\quad y = 2\sin(2\theta)$.

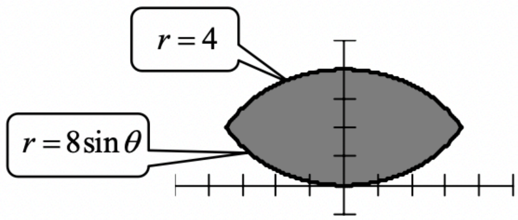
8. Find the tangents at the pole for $r = 1 + 2\cos\theta$.

9. Find the area common to the interiors of $r = 4\cos\theta$ and $r = 2$.

10. Given the function $f(x) = \ln(x+1)$:

    a. Write a power series for f showing four terms and the general term.
    b. Find the interval of convergence of this power series.
    c. Approximate $\ln(1.2)$ by using a fourth degree Taylor polynomial of f.
    d. Using your answer for part c and the alternating series remainder, give an upper and lower limit for the actual value of $\ln(1.2)$

11. The figure at the right shows a shaded region bounded by the polar curves $r = 4$ and $r = 8\sin\theta$.



    a. Find the area of the shaded region.
    b. Find the perimeter of the shaded region.
    c. Convert the two polar equations to rectangular form.
    d. Set up an integral with respect to the variable x and find the area of the shaded region.

1a. $v(4) = \langle 32, \frac{1}{4} \rangle$ b. $a(4) = \langle 8, -\frac{1}{32} \rangle$ c. $\|v(4)\| = 32.001$ d. dist. = 64.413 e. $\theta = .031$

2a. $\|v(2)\| = 12.649$ b. $\theta = 1.107$ c. dist. = 64.949 d. $v(3) = \langle 6, 27 \rangle$ e. $a(4) = \langle 2, 24 \rangle$

3a. $s(2) = \langle 6, 1 \rangle$ b. $a(2) = \langle 12, 0 \rangle$ c. $\theta = .418$ d. dist. = 4.567 or 4.568 e. never

3f. $\|v(2)\| = 8.062$ 4. $\langle 13, 72 \rangle$ 5a. $s(3) = \langle 3.394, 4.280 \text{ or } 4.281 \rangle$ b. $\|v(3)\| = .788$

5c. dist. = 1.960 or 1.961 d. $y - 4.280 = -.187(x - 3.394)$ or $y - 4.281 = -.188(x - 3.394)$

5e. $a(3) = \langle .051 \text{ or } .052, -5.936 \rangle$ 6a. 400.891 b. $y - 2 = \frac{1}{e}(x - e)$ c. concave down

6d. $y = \ln x + 1$ 7. Horiz. $(2 \pm \sqrt{2}, \pm 2)$, Vert. $(0,0), (4,0)$ 8. $\theta = \frac{2\pi}{3},\ \theta = \frac{4\pi}{3}$

9. 4.913 10a. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + \frac{(-1)^{n+1} x^n}{n} + \dots$ b. $(-1, 1]$ c. .18227

11a. 19.653 or 19.654 b. 16.755 c. $x^2 + y^2 = 16,\ x^2 + y^2 = 8y$ d. 19.653 or 19.654