name:

BC Topic 14 - Polar Arc Length

due Tuesday, February 20

Arc Length

Example 1. Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for the curve $r = 2 - 2\cos\theta$

$$\text{Arc Length} = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$A.L. = \int_0^{2\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}\, d\theta \qquad \frac{dr}{d\theta} = 2\sin\theta$$

$$= 16.000$$

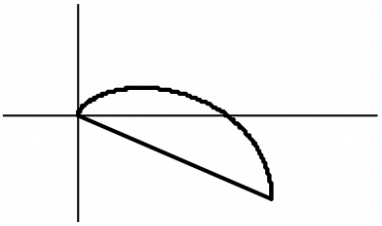
Assignment

1. Given the polar curve $r = 4\sin\theta$.
   a. Graph without using a calculator.
   b. Find the circumference using a geometry formula.
   c. Find the circumference showing a polar arc length integral setup and integrate "without" using a calculator. (We will learn trig derivatives later in AB.)

2. Graph $r = 4\cos(2\theta)$ without a calculator. Then use a calculator to find the length of the arc forming one petal.

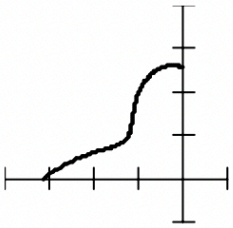
3. Use a calculator to graph $r = e^{\frac{\theta}{2}}$ on the interval $0 \le \theta \le \frac{3\pi}{2}$ and find the length of the curve.

4. The region shown is bounded by the polar curve $r = 1 - \sin\theta$ and the line $\theta = -\frac{\pi}{6}$.

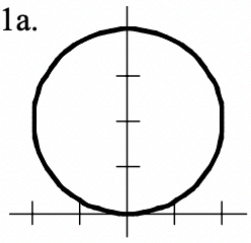


   a. Find the area of the region.
   b. Find the perimeter of the region.

15. The graph at the right shows the polar curve $r = \theta - \sin(3\theta)$ on the interval $\frac{\pi}{2} \le \theta \le \pi$.



   a. Find the area of the region bounded by the curve, the x-axis , and the y-axis.
   b. Find $\frac{dr}{d\theta}$ at $\theta = \frac{3\pi}{4}$ without using a calculator.
   c. Use your answer to part b to determine if r is increasing or decreasing on an interval containing $\theta = \frac{3\pi}{4}$.
   d. Find the value of θ on $\frac{\pi}{2} \le \theta \le \pi$ at which the curve is closest to the pole.
   e. Find the x-coordinate of the point on the curve when $\theta = \frac{3\pi}{4}$.
   f. Find $\frac{dx}{d\theta}$ at $\theta = \frac{3\pi}{4}$ using a calculator.
   g. Use your answer to part f to determine if x is increasing or decreasing on an interval containing $\theta = \frac{3\pi}{4}$.

1a. 

b. 4π 2. 9.688 3. 21.356

4a. .596 or .597 b. 3.499 or 3.500

15a. 3.756 b. $1-\frac{3}{\sqrt{2}}$ d. 2.504 or 2.505 e. -1.166 f. -.373