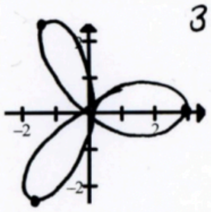
name:

# BC Topic 13 - Polar Area

due Wednesday, February 7

$$\text{Polar area} = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

Example 1. Find the area of one petal of the curve $r = 3\cos(3\theta)$.



$3\cos(3\theta) = 0$

$3\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$

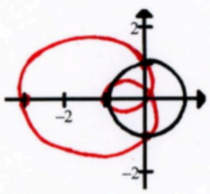
$A = 2 \cdot \frac{1}{2}\int_0^{\frac{\pi}{6}} (3\cos(3\theta))^2 \, d\theta$

$= 2.356$

Desmos visualization

## Intersections of Polar Graphs

Example 2. Find the points of intersection of the graphs of $r = 1 - 2\cos\theta$ and $r = 1$.



$1 - 2\cos\theta = 1$

$-2\cos\theta = 0$

$\cos\theta = 0$

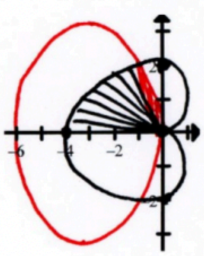
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Points: $\left(1, \frac{\pi}{2}\right)$, $\left(1, \frac{3\pi}{2}\right)$

also: $(1, \pi)$ or $(-1, 0)$ see graph

## Area between two curves

Example 3. Find the area of the region common to the two regions bounded by $r = -6\cos\theta$ and $r = 2 - 2\cos\theta$.



$-6\cos\theta = 2 - 2\cos\theta$

$-4\cos\theta = 2$

$\cos\theta = -\frac{1}{2}$

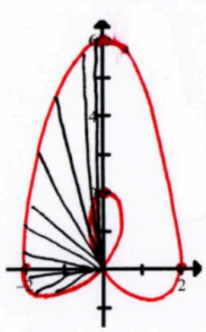
$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$A = 2 \cdot \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} (-6\cos\theta)^2 \, d\theta \; +$

$2 \cdot \frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi} (2 - 2\cos\theta)^2 \, d\theta$

$= 15.708$

Example 4. Find the area between the loops of $r = 2(1 + 2\sin\theta)$.



$2(1 + 2\sin\theta) = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$A = 2 \cdot \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{7\pi}{6}} (2(1 + 2\sin\theta))^2 \, d\theta \; -$

$2 \cdot \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} (2(1 + 2\sin\theta))^2 \, d\theta$

$= 33.351$

Without using a calculator, graph the following polar curves and find the points of intersection.

1. $r=1+\sin\theta$ and $r=1-\sin\theta$

2. $r=1+\sin\theta$ and $r=1-\cos\theta$

3. Use a calculator to graph the following curves. Then find the points of intersection.
$r=6-8\sin\theta$ and $r=2$

Graph the following polar curves without using a calculator. Set up a definite integral for the area of the indicated region. Use a calculator to evaluate the integral.

4. the interior of $r=1-\cos\theta$

5. one petal of $r=4\sin(3\theta)$

6. one petal of $r=3\cos(2\theta)$

7. the common interior of $r=3-2\cos\theta$ and $r=-3+2\cos\theta$

Use a calculator to graph the following curves. Set up a definite integral for the area of the indicated region. Use a calculator to evaluate the integral.

8. between the loops of $r=1+2\sin\theta$

9. inside $r=3\cos\theta$ and outside $r=2-\cos\theta$

10. common interior of $r=3$ and $r=6\sin(2\theta)$

11. region bounded by $r=\theta+\sin(3\theta)$ and the $x$-axis for $0\le\theta\le\pi$

12. Given the parametric equations $x=4t-1$ and $y=8t-4$, eliminate the parameter to write the corresponding rectangular equation. Sketch the curve indicating the orientation without using a calculator.

13. Without using a calculator given the parametric equations $x=3t+5$ and $y=8t^2+4$, find an equation of the line tangent to the curve when $x=2$.

14. Without using a calculator given the parametric equations $x=4\cos\theta$ and $y=8\sin\theta$, determine the concavity on an interval containing $\theta=\frac{7\pi}{6}$.

15. Given the parametric equations $x=2+2\cos\theta$ and $y=1+\sin\theta$, show work to determine the points of horizontal and vertical tangency. Graph with a calculator to see if your answers appear correct.

16. Without a calculator convert the polar point $\left(3,\frac{3\pi}{2}\right)$ to rectangular form.

17. Without a calculator convert the polar point $\left(4,\frac{2\pi}{3}\right)$ to rectangular form.

18. Without a calculator convert the rectangular point $(-5,-5)$ to polar form. Give two answers such that $0\le\theta<2\pi$.

19. Using a calculator convert the rectangular point $(-1.372\ ,\ 5.164)$ to polar form. Give two answers such that $0\le\theta<2\pi$.

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|---|---|---|---|---|
| 1. $(1,0)$, $(1,\pi)$, $(0,0)$ | 2. $\left(1+\frac{1}{\sqrt{2}},\frac{3\pi}{4}\right)$, $\left(1-\frac{1}{\sqrt{2}},\frac{7\pi}{4}\right)$, $(0,0)$ | | 4. 4.712 | |
| 5. 4.188 or 4.189 | 6. 3.534 | 7. 10.557 or 10.558 | 8. 8.337 or 8.338 | 9. 5.196 |
| 10. 22.110 or 22.111 | 11. 7.000 | 12. $y=2x-2$ | 13. $y-12=-\frac{16}{3}(x-2)$ | |
| 15. V.T: $(4,1)$, $(0,1)$ H.T: $(2,2)$, $(2,0)$ | | 17. $\left(-2,2\sqrt{3}\right)$ | 18. $\left(5\sqrt{2},\frac{5\pi}{4}\right)$, $\left(-5\sqrt{2},\frac{\pi}{4}\right)$ | |
| 19. $(5.343,1.830)$, $(-5.343,4.972)$ | | | | |