name:

**BC Topic 12 - Parametric Equations**
due Friday, January 26

Rectangular coordinates: $(x, y)$

Polar coordinates: $(r, \theta)$

Parametric coordinates: $(x, y, t)$ ← **This one looks rectangular but moves through time.**

**Desmos can show this with animation.** →

**On paper, you will just see a timelaspe.**

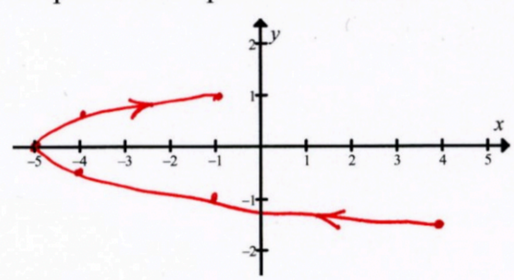
Example 1. Plot points to sketch the curve described by the parametric equations. Mark the orientation on the curve.

$x = t^2 - 5$

$y = \frac{t}{2}$

$-3 \le t \le 2$

| $t$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|
| $x$ | $4$ | $-1$ | $-4$ | $-5$ | $-4$ | $-1$ |
| $y$ | $-\frac{3}{2}$ | $-1$ | $-\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $1$ |



Example 2. Change the following to rectangular form by eliminating the parameter. Then graph.

$x = \frac{1}{\sqrt{t+1}}$ and $y = \frac{t}{t+1}$, $t > -1$

$x^2 = \frac{1}{t+1}$

$\frac{1}{x^2} = t+1$

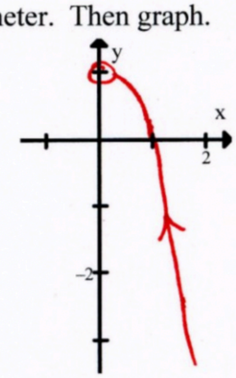
$\frac{1}{x^2} - 1 = t$

$y = \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} - 1 + 1}$

$y = \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2}} \cdot \frac{x^2}{x^2}$

$y = \frac{1 - x^2}{1}$

$y = 1 - x^2$, $x > 0$



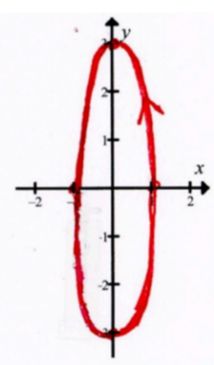
Example 3. Eliminate the parameter to sketch the curve.

$x = \cos\theta$ and $y = 3\sin\theta$, $0 \le \theta \le 2\pi$

$\cos^2\theta + \sin^2\theta = 1$

$x^2 + \left(\frac{y}{3}\right)^2 = 1$

$x^2 + \frac{y^2}{9} = 1$



Example 5. Find the slope and concavity of $x = \sqrt{t}$ and $y = \frac{1}{4}t^2 - 1$, $t \geq 0$ at the point $(2,3)$.

$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$

$\frac{dy}{dt} = \frac{1}{2}t$

$\frac{dy}{dx} = \frac{\frac{1}{2}t}{\frac{1}{2}t^{-\frac{1}{2}}}$

$= t^{3/2}$

$\text{slope} = \left.\frac{dy}{dx}\right|_{(2,3)}$

$= \left.\frac{dy}{dx}\right|_{t=4}$

$= 4^{3/2}$

$= 8$

$\frac{d^2y}{dx^2} = \frac{\frac{3}{2}t^{\frac{1}{2}}}{\frac{1}{2}t^{-\frac{1}{2}}}$

$= 3t$

$\left.\frac{d^2y}{dx^2}\right|_{t=4} = 12$

The curve is conc. upward.

Example 6. Write an equation of a tangent line to the curve defined by $x = t - 1$ and $y = \frac{1}{t} + 1$ at the point when $t = 1$.

$x(1) = 0$

$y(1) = 2$

point: $(0, 2)$

$\frac{dx}{dt} = 1$

$\frac{dy}{dt} = -t^{-2}$

$\frac{dy}{dx} = -t^{-2}$

$\left.\frac{dy}{dx}\right|_{t=1} = -1$

Tan. Line:

$y - 2 = -1(x - 0)$

**Arc Length:** If a curve is smooth and does not intersect itself the length of an arc is given by

$$\boxed{\text{arc length} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt}$$

Example 7. Using the parametric equations from example 6, find the arc length on the interval $1 \leq t \leq 3$.

$A.L. = \int_1^3 \sqrt{1^2 + (-t^{-2})^2}\, dt$

$= 2.147$

**Use a TI-84 or TI-Nspire for all calculator questions.**
**Desmos will be unavailable on the quiz.**

Using these parametric equations, eliminate the parameter to write the corresponding rectangular equation. Sketch the curve indicating the orientation without using a calculator.

1. $x=2t-3,\ y=\frac{2}{3}t+4$ 2. $x=t^3,\ y=t^2$ 3. $x=\sqrt{t},\ y=4-t$ 4. $x=t^4,\ y=4\ln t$

5. Use a calculator set in parametric mode to graph the curve represented by the parametric equations $x=-3+4\cos\theta$ and $y=1+2\sin\theta$. Then eliminate the parameter.

6. Given the parametric equations $x=-2t+1$ and $y=t^3+3$:

a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

b. Find an equation of the tangent line when $t=1$.

c. Use concavity to determine if the tangent line is above the curve or below the curve.

Find all points at which each curve has horizontal and vertical tangents.

9. $x=2t+1,\ y=t^2$ 10. $x=t^2+1,\ y=t^2+4t$

11. $x=t^2-t+3,\ y=4t^3-12t$

Given the parametric equations $x=-3t-5$ and $y=t^3-12t+3$ (without using a calculator):

13. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of $t$.

14. Use the second derivative test to determine if the curve has a local maximum, a local minimum, or neither when $t=2$.

15. Use the second derivative test to determine if the curve has a local maximum, a local minimum, or neither at the point $(1,19)$.

16. Use the second derivative test to determine if the curve has a local maximum, a local minimum, or neither at the point $(-8,-8)$.

Show an integral setup and find the length of each arc on the given interval with a calculator.

17. $x=3t-t^2,\ y=4t^{\frac{3}{2}}$ $1\le t\le 2$

18. $x=t+\cos t,\ y=t-\sin t$ $0\le t\le \pi$

19. $x=\arccos t,\ y=\ln\sqrt{1+t^2}$ $0\le t\le \frac{1}{2}$

20. Find the length of the arc between the two $y$-intercepts of $x=t^2-1$ and $y=2t$.

1. $y=\frac{1}{3}x+5$ 2. $y=x^{\frac{2}{3}}$ 4. $y=\ln x$ 5. $\left(\frac{x+3}{4}\right)^2+\left(\frac{y-1}{2}\right)^2=1$

6a. $\frac{dy}{dx}=-\frac{3}{2}t^2$, $\frac{d^2y}{dx^2}=\frac{3}{2}t$ b. $y-4=-\frac{3}{2}(x+1)$ c. below

9. H.T: (1,0) V.T: none 10. H.T: $(5,-4)$ V.T: (1,0)

11. H.T: (3,-8), (5,8) V.T: $\left(\frac{11}{4},-\frac{11}{2}\right)$

13. $\frac{dy}{dx}=-t^2+4$, $\frac{d^2y}{dx^2}=\frac{2}{3}t$ 14. min. 15. max. 17. 7.336 or 7.337

18. 3.678 19. .538 20. 4.591