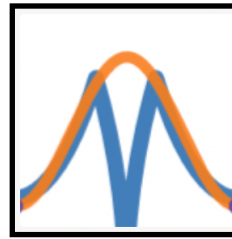# BC Topic 10 - Polar Graphs
due Wednesday, December 20

Polar graphs require no calculus.
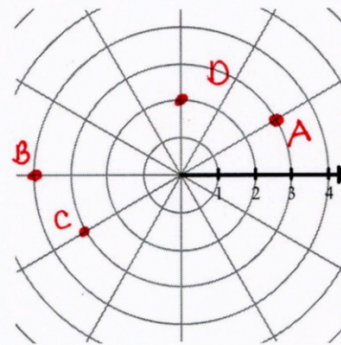
Log in at student.desmos.com to find this intro activity:

Slides 1 and 2 will give you a feel for these graphs.
Slides 3 to 20 will be unlocked during class December 14.
(We'll do a group competition thing.)

Plotting points in polar form
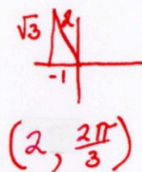1. Use the polar grid to plot these polar points.

A $\left(3, \frac{\pi}{6}\right)$ B $(4, \pi)$

C $\left(3, -\frac{5\pi}{6}\right)$ D $\left(-2, \frac{3\pi}{2}\right)$

Point conversions
2. Write the rectangular point $\left(-1, \sqrt{3}\right)$ in polar form such that:

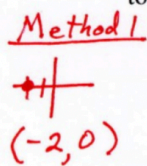a. $r > 0, \ \theta > 0$ b. $r > 0, \ \theta < 0$ c. $r < 0, \ \theta > 0$ d. $r < 0, \ \theta < 0$

$\left(2, \frac{2\pi}{3}\right)$ $\left(2, -\frac{4\pi}{3}\right)$ $\left(-2, \frac{5\pi}{3}\right)$ $\left(-2, -\frac{\pi}{3}\right)$

Conversion Equations
$x^2 + y^2 = r^2$ $x = r\cos\theta$
$\tan\theta = \frac{y}{x}$ $y = r\sin\theta$
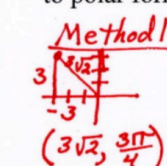
3. Change the point $(2, \pi)$ to rectangular form.

Method 1
$(-2, 0)$

Method 2
$x = 2\cos\pi = -2$
$y = 2\sin\pi = 0$
$(-2, 0)$

4. Change the point $(-3, 3)$ to polar form.

Method 1
$\left(3\sqrt{2}, \frac{3\pi}{4}\right)$
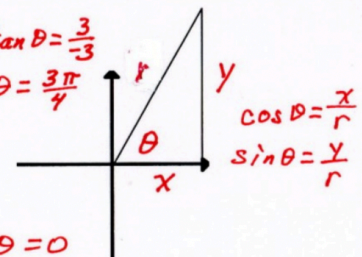
Method 2
$(-3)^2 + 3^2 = r^2$
$2\sqrt{3} = r$
$\left(2\sqrt{3}, \frac{3\pi}{4}\right)$

$\tan\theta = \frac{3}{-3}$
$\theta = \frac{3\pi}{4}$

$\cos\theta = \frac{x}{r}$
$\sin\theta = \frac{y}{r}$

Equation conversions (rectangular to polar)

5. $y = 4$
$r\sin\theta = 4$
$r = \frac{4}{\sin\theta}$

6. $3x - y + 2 = 0$
$3r\cos\theta - r\sin\theta + 2 = 0$
$r(3\cos\theta - \sin\theta) = -2$
$r = \frac{-2}{3\cos\theta - \sin\theta}$

7. $x^2 + y^2 - 2x = 0$
$r^2 - 2r\cos\theta = 0$
$r(r - 2\cos\theta) = 0$
$r \neq 0, \quad r = 2\cos\theta$

Equation conversions (polar to rectangular)

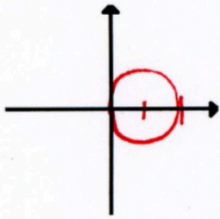8. $r = -2$

$$r^2 = 4$$
$$x^2 + y^2 = 4$$

9. $r = 3\cos\theta$

$$r^2 = 3r\cos\theta$$
$$x^2 + y^2 = 3x$$

10. $r = 2\csc\theta$

$$r = \frac{2}{\sin\theta}$$
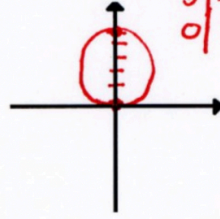$$r\sin\theta = 2$$
$$y = 2$$

Sketching polar graphs (use a calculator on those in **bold**)
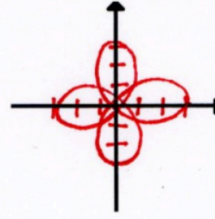
11. Circles

a. $r = 2\cos\theta$

b. $r = 5\sin\theta$

r	θ
0	0
5	$\frac{\pi}{2}$
0	π

12. Rose petal curves

a. $r = 3\cos(2\theta)$

b. $r = 4\sin(3\theta)$

r	θ
4	0
4	$\frac{\pi}{6}$
0	$\frac{\pi}{3}$
-4	$\frac{\pi}{2}$
0	$\frac{2\pi}{3}$
4	$\frac{5\pi}{6}$
0	π

13. Limaçons

a. $r = 2 - 3\cos\theta$

r	θ
3	0
6	$\frac{\pi}{2}$
3	π
0	$\frac{3\pi}{2}$

b. $r = 3 + 3\sin\theta$
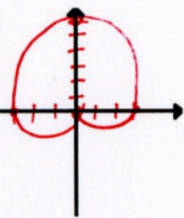
c. $r = 2 - 2\cos\theta$
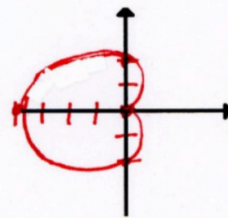
r	θ
0	0
2	$\frac{\pi}{2}$
4	π
2	$\frac{3\pi}{2}$

14. Lemniscate

$$r^2 = 9\sin(2\theta)$$
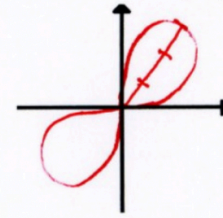
Sketching polar graphs:

Circles: $\begin{aligned} r &= d\cos\theta \quad \text{(x-axis symmetry)} \\ r &= d\sin\theta \quad \text{(y-axis symmetry)} \end{aligned}\Big\}$ d is the diameter

Rose petal curves: $\begin{aligned} r &= a\cos(n\theta) \quad \text{(x-axis symmetry)} \\ r &= a\sin(n\theta) \quad \text{(y-axis symmetry)} \end{aligned}\Big\}$ $\begin{aligned} &a \text{ is the maximum } r, \\ &n \text{ petals if } n \text{ is odd,} \\ &2n \text{ petals if } n \text{ is even} \end{aligned}$

Limaçons: $\begin{aligned} r &= a \pm b\cos\theta \quad \text{(x-axis symmetry)} \\ r &= a \pm b\sin\theta \quad \text{(y-axis symmetry)} \end{aligned}\Big\}$ $\begin{aligned} &\text{may have inner loop} \\ &\text{may or may not include the pole} \end{aligned}$
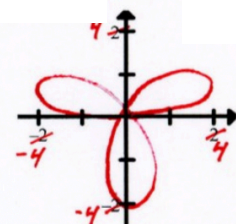

Desmos visualization

Tangent lines at the pole

17. Find the equations of the lines tangent to $r = 4\sin(3\theta)$ at the pole.

$$4\sin(3\theta) = 0$$
$$3\theta = 0, \pi, 2\pi, \ldots,$$
$$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

Without using a calculator, accurately plot each of the following polar coordinate points on a separate graph. Give the rectangular coordinates of the point.

1. $\left(5, \frac{\pi}{2}\right)$
2. $\left(-3\sqrt{2}, \frac{3\pi}{4}\right)$
3. $\left(4, -\frac{\pi}{3}\right)$
4. $\left(-1, \frac{7\pi}{6}\right)$

Without using a calculator, plot each of the following rectangular points and give two sets of polar coordinates for $0 \le \theta < 2\pi$.

5. $(5, -5)$
6. $\left(-1, \sqrt{3}\right)$
7. $(-5, 0)$
8. $\left(-5, -5\sqrt{3}\right)$

9. Use a calculator to give rectangular coordinates for the polar point $(-7.2, 4.5)$.
10. Use a calculator to give two sets of polar coordinates ($0 \le \theta < 2\pi$) for the rectangular point $(-2, 5)$.

Match each of the following equations with one of the descriptions given without using a calculator.

11. $r = 3\sin(2\theta)$
12. $r = 4\cos\theta$
13. $r\cos\theta = 4$
14. $r = 4 + 2\cos\theta$

a. a circle with y-axis symmetry
b. a four petal rose
c. a vertical line
d. a limaçon with y-axis symmetry
e. a circle with x-axis symmetry
f. a horizontal line.
g. a limaçon with x-axis symmetry

Convert the following rectangular equations to polar (solve for r) and sketch the graph.

15. $y = 5$
16. $2x + y + 5 = 0$
17. $y^2 = 2x$

Convert the following polar equations to rectangular and sketch the graph.

18. $r = 5$
19. $\theta = \frac{3\pi}{4}$
20. $r = 3\sec\theta$

Use a calculator to graph. Determine if the interval $0 \le \theta < 2\pi$ produces a complete graph.

21. $r = 5\sin\left(\frac{3\theta}{2}\right)$
22. $r = 5 - 6\cos\theta$
23. $r = \theta$

Without using a calculator graph each equation and find equations of each tangent at the pole.

27. $r = 5\sin\theta$
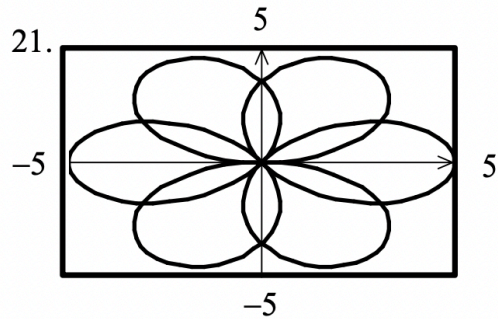28. $r = 3 - 3\cos\theta$
29. $r = 4\cos(3\theta)$

Selected Answers:

show graphs for 1-8   1. (0,5) 2. (3,-3)   3. $\left(2,-2\sqrt{3}\right)$ 5. $\left(5\sqrt{2},\frac{7\pi}{4}\right)$, $\left(-5\sqrt{2},\frac{3\pi}{4}\right)$

7. $\left(5,\pi\right)$, $\left(-5,0\right)$ 8. $\left(10,\frac{4\pi}{3}\right)$, $\left(-10,\frac{\pi}{3}\right)$ 9. (1.517 or 1.518 , 7.038)

11. b   13. c   15. $r = 5\csc\theta$ 16. $r = \dfrac{-5}{2\cos\theta + \sin\theta}$ 17. $r = \dfrac{2\cos\theta}{\sin^2\theta}$

19. $y = -x$ 20. $x = 3$

21.

$0 \leq \theta < 4\pi$ shows a complete graph.

27. $\theta = 0$ 29. $\theta = \dfrac{\pi}{6},\ \dfrac{\pi}{2},\ \dfrac{5\pi}{6}$