BC Calc Memorization Sheet

Derivatives

Prod. $\frac{d}{dx}(uv) = uv' + vu'$

Quot. $\frac{d}{dx}\frac{t}{b} = \frac{bt' - tb'}{b^2}$

Chain $\frac{d}{dx} f(u) = f'(u)\, u'$

or

$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$

$\frac{d}{dx} x^n = nx^{n-1}$

$\frac{d}{dx} \ln x = \frac{1}{x}$, $\frac{d}{dx} \ln u = \frac{u'}{u}$

$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

$\frac{d}{dx} e^x = e^x$, $\frac{d}{dx} a^x = a^x \ln a$

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \cot x = -\csc^2 x$

$\frac{d}{dx} \sec x = \sec x \tan x$

$\frac{d}{dx} \csc x = -\csc x \cot x$

$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{u'}{u} dx = \ln|u| + C$

$\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \cos x\, dx = \sin x + C$

$\int \sin x\, dx = -\cos x + C$

$\int \sec^2 x\, dx = \tan x + C$

$\int \tan x\, dx = -\ln|\cos x| + C$

$\int \cot x\, dx = \ln|\sin x| + C$

$\int \sec x \tan x\, dx = \sec x + C$

$\int \csc^2 x\, dx = -\cot x + C$

$\int \csc x \cot x\, dx = -\csc x + C$

$\int \frac{u'}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \frac{u'}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$

Parts $\int u\, dv = uv - \int v\, du$

$\int \ln x\, dx = x \ln x - x + C$

Trig Identities

$\tan x = \frac{\sin x}{\cos x}$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x = \sec^2 x - 1$

Volume

Disc

$V = \pi \int_a^b r^2\, dx$

Washer

$V = \pi \int_a^b (R^2 - r^2)\, dx$

Shell

$V = 2\pi \int_a^b rh\, dx$

Cross Section

$V = \int_a^b A\, dx$

Second Fundamental Theorem

$\frac{d}{dx} \int_u^v f(t)\, dt = f(v)v' - f(u)u'$

Area of Trapezoid $A = \frac{1}{2} w(h_1 + h_2)$

Trapezoidal Rule

$A \approx \frac{1}{2} w(h_1 + 2h_2 + 2h_3 + \ldots + 2h_n + h_{n+1})$

Riemann Sum (add areas of rectangles)

Alt. Series Error: error $\leq |a_{n+1}|$ (the next term)

Lagrange Error:

error $\leq \left| \frac{f^{(n+1)}(z)(x - c)^{n+1}}{(n+1)!} \right|$ where $|f^{(n+1)}(z)|$

is a maximum between x and c

Logistic

$\frac{dP}{dt} = kP(M - P)$

$P = \frac{M}{1 + Ce^{-kMt}}$

M = carrying capacity

Elementary Series

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$

$\ln x = (x - 1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots + \frac{(-1)^{n+1}(x-1)^n}{n} + \cdots$

Taylor Series

$f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \frac{f'''(c)}{3!}(x - c)^3 + \cdots + \frac{f^{(n)}(c)}{n!}(x - c)^n + \cdots$

Maclaurin Series (Taylor series with $c = 0$)

Euler's

x	y	$\frac{dy}{dx}$	$\Delta y = m\Delta x$

Average Rate of Change: $\text{AROC} = \dfrac{f(b) - f(a)}{b - a}$ (slope between two points)

Inst. Rate of Change: $\text{IROC} = f'(c)$ (slope at a single point)

Mean Value Thm.: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ (find c where $m_{\text{sec}} = m_{\text{tan}}$)

Average Value of a Function: $f_{avg} = \dfrac{\int_a^b f(x)dx}{b-a}$ $\left(\text{avg. height} = \dfrac{\text{area}}{\text{width}}\right)$

Intermediate Value Thm. A function f that is continuous on $[a,b]$ takes on every y-value between $f(a)$ and $f(b)$.

Arc Length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx = \underbrace{\int_{t_1}^{t_2} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt}_{\text{parametric, vectors}} = \underbrace{\int_{\theta_1}^{\theta_2} \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta}_{\text{polar}}$

Speed $= |v(t)| = \underbrace{\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}}_{\text{vectors}}$ **Total Dist.** $= \int_{t_1}^{t_2} |v(t)|\, dt = \int_{t_1}^{t_2} \underbrace{\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}\, dt}_{\text{vectors}}$

Polar Area $= \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\, d\theta$ **Parametric Derivatives:** $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$, $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Polar Conversions: $r^2 = x^2 + y^2$, $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \arctan\frac{y}{x}$

Tests for Convergence/Divergence

n^{th} term test	div. if $\lim\limits_{n\to\infty} a_n \neq 0$ (cannot be used to show convergence)
Geom. series test	$\sum\limits_{n=0}^{\infty} ar^n$ $\lvert r\rvert < 1 \to$ conv. , $\lvert r\rvert \geq 1 \to$ div. , $S = \dfrac{a}{1-r}$
p-series	$\sum\limits_{n=1}^{\infty} \dfrac{1}{n^p}$ $p > 1 \to$ conv. , $p \leq 1 \to$ div.
Alternating series	decr. terms and $\lim\limits_{n\to\infty} a_n = 0 \to$ conv.
Integral test	$a_n = f(x)$ $\sum\limits_{n=1}^{\infty} a_n$ conv. if $\int_1^{\infty} f(x)\, dx$ conv., $\sum\limits_{n=1}^{\infty} a_n$ div. if $\int_1^{\infty} f(x)\, dx$ div.
Ratio test	$\lim\limits_{n\to\infty}\left\lvert\dfrac{a_{n+1}}{a_n}\right\rvert < 1 \to$ conv. , $\lim\limits_{n\to\infty}\left\lvert\dfrac{a_{n+1}}{a_n}\right\rvert > 1 \to$ div. , (inconclusive if $\lim\limits_{n\to\infty}\left\lvert\dfrac{a_{n+1}}{a_n}\right\rvert = 1$) (works well for factorials and exponentials)
Direct Comparison	a series with terms **smaller** than a known convergent series also converges a series with terms **larger** than a known divergent series also diverges
Limit Comparison	if $\lim\limits_{n\to\infty}\dfrac{a_n}{b_n}$ is finite and positive both series converge or both diverge (use with "messy" algebraic series, usually compared to a p-series)