Assignment 1 Solutions 1997 M.C. (selected Problems)

① $\int_1^2 (4x^3 - 6x)dx$
$(x^4 - 3x^2)\Big|_1^2$
$16 - 12 - (1 - 3)$
$6 \quad \boxed{C}$

② $f(x) = x\sqrt{2x-3}$
$f(x) = x(2x-3)^{\frac{1}{2}}$
$f'(x) = x \cdot \frac{1}{2}(2x-3)^{-\frac{1}{2}} \cdot 2 + (2x-3)^{\frac{1}{2}}$
$= \frac{x}{\sqrt{2x-3}} + \sqrt{2x-3}$
$= \frac{x + 2x - 3}{\sqrt{2x-3}}$
$= \frac{3x-3}{\sqrt{2x-3}} \quad \boxed{A}$

⑤ $y = 3x^4 - 16x^3 + 24x^2 + 48$
$y' = 12x^3 - 48x^2 + 48x$
$y'' = 36x^2 - 96x + 48$
$y'' = 12(3x^2 - 8x + 4)$
$y'' = 12(3x-2)(x-2)$
y'' + | − | +
$\frac{2}{3}$ 2
conc. dn. for $\frac{2}{3} < x < 2 \quad \boxed{E}$

⑥ $\frac{1}{2}\int e^{\frac{t}{2}}dt =$
$2 \cdot \frac{1}{2}\int \textcircled{\tfrac{1}{2}} e^{\frac{1}{2}t}\,dt$
$e^{\frac{1}{2}t} + C \quad \boxed{C}$

⑦ $\frac{d}{dx}\cos^2(x^3) =$
$\frac{d}{dx}(\cos x^3)^2 =$
$2(\cos x^3)\cdot(-\sin x^3)\cdot 3x^2$
$-6x^2 \cos x^3 \sin x^3 \quad \boxed{D}$

⑩ $y = \cos(2x)$
$y\left(\frac{\pi}{4}\right) = \cos\left(2 \cdot \frac{\pi}{4}\right)$
$= \cos\frac{\pi}{2}$
$= 0$
Pt. of tangency is $\left(\frac{\pi}{4}, 0\right)$
$y' = -\sin(2x)\cdot 2$
$y'\left(\frac{\pi}{4}\right) = -2\sin\frac{\pi}{2}$
$= -2$
slope of tangent is -2
$y - 0 = -2\left(x - \frac{\pi}{4}\right) \quad \boxed{E}$

⑫ parallel to $2x - 4y =$
$y = \frac{1}{2}x - \frac{3}{4}$
slope $= \frac{1}{2}$
$y = \frac{1}{2}x^2$
$y' = x$
$x = \frac{1}{2}$
$y\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}\right)^2$
$= \frac{1}{8}$
Point: $\left(\frac{1}{2}, \frac{1}{8}\right) \quad \boxed{B}$

⑮ $\lim_{x \to a} f(x)$ is the y-value of the hole $=$

$\lim_{x \to b} f(x)$ DNE because it is different from the left and right

$\boxed{B}$

---

⑰ $x^2 + y^2 = 25$
$2x + 2yy' = 0$
$2yy' = -2x$
$y' = -\frac{x}{y}$
$y'' = \frac{y(-1) - (-x)y'}{y^2}$
$y'' = \frac{-y + x\left(-\frac{x}{y}\right)}{y^2}$
$y''(4,3) = \frac{-3 + 4\left(-\frac{4}{3}\right)}{9}$
$= -\frac{25}{27} \quad \boxed{A}$

⑲ $f(x) = \ln|x^2 - 1|$
$f'(x) = \frac{2x}{x^2-1} \quad \boxed{D}$

(76) $f(x) = \frac{e^{2x}}{2x}$
$f'(x) = \frac{2x \cdot 2e^{2x} - e^{2x} \cdot 2}{(2x)^2}$
$= \frac{2e^{2x}(2x-1)}{4x^2}$
$\boxed{E} \quad = \frac{e^{2x}(2x-1)}{2x^2}$

(79) $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$ is the limit definition of the derivative with x replaced by 2.
In other words
$f'(2) = 5$
If the derivative exists f is differentiable and also continuous. $\boxed{C}$

(80) $f(x) = 2e^{4x^2}$
$f'(x) = 2e^{4x^2} \cdot 8x$
$f'(x) = 16xe^{4x^2}$
$16xe^{4x^2} = 3$
Solving with acalculator:
$Y_1 = 16xe^{4x^2} \quad Y_2 = 3$
$x: [0, .6] \quad y: [-2, 5]$
find intersection
$x = .1675\ldots \quad \boxed{A}$

(82) $P = xy = x(2x-8)$
$P = 2x^2 - 8x$
$P' = 4x - 8 \qquad \frac{CN}{x = 2}$
P' ← − | + →
2
min @ $x =$
Prod $= 2 \cdot 2^2 - 8 \cdot 2 = -8$

(83) $\cos x = x$
$x = .739$
$A = \int_0^{.739}(\cos x - x)dx$
$= .4004\ldots \quad \boxed{C}$

(85) $f'(x) = e^x - 3x^2 = 0$
CN/$x = -.459, .910, 3.7$
f' ← − | + | − | + →
$-.459 \quad .910 \quad 3.733$
rel. max. at
$x = .910$
$\boxed{C}$