LESSON 9-7 FINAL INTEGRATION TECHNIQUES

Adding and subtracting terms in the numerator or multiplying both the numerator and denominator by a common factor may make integration possible.

Examples: Integrate.

1. $\int \frac{2x+2-2}{x^2+2x+1}\,dx$

$\int \frac{2x+2}{x^2+2x+1}\,dx - \int \frac{2}{(x+1)^2}\,dx$

$= \ln|x^2+2x+1| - 2\int (x+1)^{-2}\,dx$

$= \ln|x^2+2x+1| + 2(x+1)^{-1} + C$

2. $\int \frac{1}{e^x+1}\,dx$

Method 1:

$\int \frac{1+e^x-e^x}{e^x+1}\,dx$

$= \int 1\,dx - \int \frac{e^x}{e^x+1}\,dx$

$= x - \ln|e^x+1| + C$

Method 2: Mult. by $\frac{e^{-x}}{e^{-x}}$

$-\int \frac{-e^{-x}}{1+e^{-x}}\,dx$

$= -\ln|1+e^{-x}| + C$

A trig substitution (or use of a trig identity) may be necessary for some problems. The common trig substitutions are those for $\tan^2 x$ and $\cot^2 x$.

$\tan^2 x = \sec^2 x - 1$ and $\cot^2 x = \csc^2 x - 1$.

Example 3: Integrate $\int \left(\tan^2(2x) + \cot^2(3x)\right)dx$

$\int \left(\sec^2(2x) - 1 + \csc^2(3x) - 1\right)dx$

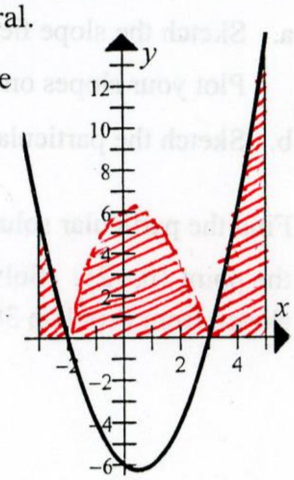
$= \frac{1}{2}\tan(2x) - x - \frac{1}{3}\cot(3x) - x + C$

Definite integrals involving absolute value expressions can often be evaluated by using the ideas of "signed area" and/or symmetry. However, more complicated integrals involving absolute value may require you to "split" the integral.

Example 4: Set up integrals (which do not contain absolute values) that could be used to integrate $\int_{-3}^{5} \left|x^2 - x - 6\right| dx$

$(x-3)(x+2)$

You do not need to actually integrate.



$\int_{-3}^{-2}(x^2-x-6)\,dx - \int_{-2}^{3}(x^2-x-6)\,dx + \int_{3}^{5}(x^2-x-6)\,dx$

There are many techniques for integration. This course deals with only the most common of them. When integrating, explore options until you find one that works. Practice fitting integrands into the basic integration formulas that you have memorized (or will memorize).

Examples: Integrate.

5. $\int \frac{\sin\left(e^{(-3x)}\right) - e^{x}}{e^{3x}}\,dx$

$-\frac{1}{3}\int \sin(e^{-3x})(-3)e^{-3x}\,dx - \left(-\frac{1}{2}\right)\int(-2)e^{-2x}\,dx$

$\frac{1}{3}\cos(e^{-3x}) + \frac{1}{2}e^{-2x} + C$

6. $\int \frac{\cos x\left(\ln(\sin x)\right)^2}{\sin x}\,dx$

$\int (\ln(\sin x))^2 \frac{\cos x}{\sin x}\,dx$

$\frac{(\ln(\sin x))^3}{3} + C$

Remember that if integrals involve radicals, and you can't seem to fit the integrand into any basic integration form, you should try u-substitution.

Example 7: Integrate $\int \frac{x+2}{\sqrt{x+1}}\,dx = \int \frac{u-1+2}{u^{\frac{1}{2}}}\,du$

$u = x+1$

$du = dx$

$u-1 = x$

$= \int \left(u^{\frac{1}{2}} + u^{-\frac{1}{2}}\right)du$

$= \frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}} + C$

$= \frac{2}{3}(x+1)^{\frac{3}{2}} + 2(x+1)^{\frac{1}{2}} + C$

ASSIGNMENT 9-7

Integrate in Problems 1-16.

1. $\int \frac{2x}{x^2-4x+4}\,dx$ Hint: See Example 1
2. $\int \frac{\sin^2(2x)-2}{\cos^2(2x)}\,dx$ Hint: Split
3. $\int t^2\tan\left(t^3\right)dt$
4. $\int \frac{2x}{9+x^2}\,dx$
5. $\int \frac{2\,dx}{9+x^2}$
6. $\int \frac{(e^{2y}-1)^2}{e^y}\,dy$
7. $\int \frac{2}{1-e^{2y}}\,dy$ Hint: See Example 2
8. $\int \frac{dx}{x\sqrt{\ln x}}$
9. $\int \frac{3-4t}{t^2+9}\,dt$ Hint: Split
10. $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$ Hint: Let the denominator equal u
11. $\int \frac{\arcsin\frac{x}{2}}{\sqrt{4-x^2}}\,dx$ Hint: Let the numerator equal u
12. $\int \frac{\sqrt{x-1}}{x}\,dx$ Hint: Let $u = \sqrt{x-1}$

Differentiate in Problems 13-16.

13. $y = \ln\left(\arctan(2x)\right)$
14. $f(x) = \ln(x)\arctan(2x)$
15. $g(x) = \left(\ln\left(\arcsin x^2\right)\right)^3$
16. For $\sin\left(t^2\right) = ye^t$, find $\frac{dy}{dt}$