## LESSON 9-6 REVIEW OF BASIC INTEGRATION FORMULAS

A summary listing of all of the integration formulas that you will be expected to know for the AP Exam appears on page 206. The list may seem overwhelming now, but if you think about each formula as you use it, and if you work on memorizing the formulas by spending just a little time with them each day, you will be fine on "AP Test Day."

Memorization alone will not be enough, however. Integration is not as straightforward as differentiation. Differentiation is sort of a "follow the recipe(s)" approach. With integration, you must distinguish among "look-alike" problems, and you must often make adjustments to fit an integral into one of the memorized forms.

Success with integration requires:

1. Memorization of the formulas and the ability to understand and use them.
2. Recognition of which formula to use for a given problem.
3. Ability to use algebraic techniques and/or substitution to put problems into forms which fit the formulas.
4. Lots of practice.

Example 1: Integrate these four "look-alike" integrals. Although they have similar appearances, they will require you to use three completely different integration formulas.

arctan type a. $\int \frac{dx}{1+x^2} = \int \frac{1}{1+x^2}dx = \arctan x + C$

ln type b. $\int \frac{dx}{1+x} = \int \frac{1}{1+x}dx = \ln|1+x| + C$

ln type c. $\frac{1}{2}\int \frac{2x\,dx}{1+x^2} = \frac{1}{2}\ln|1+x^2| + C$

reverse chain (power) type d. $\int \frac{dx}{(1+x)^2} = \int (1+x)^{-2}dx = -(1+x)^{-1} + C$ or $\frac{-1}{1+x} + C$

Example 2: Integrate the following two "look-alikes."

ln type a. $\int \frac{dx}{x\ln x} = \int \frac{\frac{1}{x}}{\ln x}dx = \ln|\ln x| + C$

reverse chain b. $\int \frac{\ln x}{x}dx = \int (\ln x)^1 \cdot \frac{1}{x}dx = \frac{1}{2}(\ln x)^2 + C$

These are the simplest algebraic techniques that we have used to fit integrals into forms which appear in the basic integration formulas.

1. Multiplication (expansion).
2. Term by term division (dividing by a monomial).
3. Long division (dividing by a polynomial). Use this technique when the degree of the numerator $\geq$ the degree of the denominator.
4. Splitting (separating) the numerator.

The more complicated and "harder to see" techniques will be discussed in the final lesson – Lesson 6-8.

Examples: Integrate.

3. $\int (e^{2x}-2)^2\,dx = \int (e^{4x} - 4e^{2x} + 4)\,dx$

$= \frac{1}{4}e^{4x} - 2e^{2x} + 4x + C$

4. $\int \frac{x - 2\sqrt{x} + 1}{\sqrt{x}}\,dx = \int (x^{\frac{1}{2}} - 2 + x^{-\frac{1}{2}})\,dx$

$= \frac{2}{3}x^{\frac{3}{2}} - 2x + 2x^{\frac{1}{2}} + C$

5. $\int \frac{x^2 - x - 1}{x^2 + x}\,dx$

$$\begin{array}{r} 1 - \frac{2x+1}{x^2+x} \\ x^2+x \overline{)\, x^2 - x - 1} \\ \underline{x^2 + x} \\ -2x - 1 \end{array}$$

$\int 1\,dx - \int \frac{2x+1}{x^2+x}\,dx$

$= x - \ln|x^2+x| + C$

6. $\int \frac{3 - x}{\sqrt{1 - x^2}}\,dx$

$\int \frac{3}{\sqrt{1-x^2}}\,dx + \int \frac{-x}{\sqrt{1-x^2}}\,dx$

$= 3\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}}(-2x)\,dx$

$= 3\arcsin x + (1-x^2)^{\frac{1}{2}} + C$

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**Note**: You separated the numerator to create basic integration forms in Example 6.

Do not make this error: $\int \frac{1}{x^2+1}\,dx = \int \frac{1}{x^2}\,dx + \int 1\,dx$. You can not separate a denominator. How would you integrate this problem? as an arctan.

Examples: Integrate. arctan type

$\int e^u u'\,dx$ type

7. $\int \sin(2x)e^{\cos(2x)}\,dx$

$u = \cos(2x)$

$u' = -\sin(2x)\cdot 2$

$= -\frac{1}{2}\int e^{\cos(2x)}(-2\sin(2x))\,dx = -\frac{1}{2}e^{\cos(2x)} + C$

8. $\int \frac{\sec^2 x}{\tan^2 x + 9}\,dx = \int \frac{\sec^2 x}{3^2 + (\tan x)^2}\,dx = \frac{1}{3}\arctan\left(\frac{\tan x}{3}\right) + C$

$a = 3$

$u = \tan x$

$u' = \sec^2 x$

**ASSIGNMENT 9-6**

Integrate in Problems 1-15.

1. $\int \frac{2x - \sqrt{x} + 3}{\sqrt{x}}\,dx$
2. $\int (2x+1)^6\,dx$
3. $\int \left(3t^2 - 1\right)^2\,dt$
4. $\int \frac{\sqrt{\ln y}}{y}\,dy$
5. $\int \frac{\sec^2\theta}{1 + \tan\theta}\,d\theta$
6. $\int \frac{\sec^2\theta}{1 + \tan^2\theta}\,d\theta$
7. $\int \frac{\sec^2\theta}{(1 + \tan\theta)^2}\,d\theta$
8. $\int \frac{\tan\theta\sec^2\theta}{1 + \tan^2\theta}\,d\theta$
9. $\int \frac{2x^2 - 4}{x + 1}\,dx$
10. $\int e^{\cos(3u)}\sin(3u)\,du$
11. $\int x^2\left(\sin x^3\right)^{\frac{3}{2}}\cos x^3\,dx$
12. $\int \frac{e^{5x} - e^x + 2}{e^{2x}}\,dx$
13. $\int \frac{4x}{\sqrt{1 - x^4}}\,dx$
14. $\int \frac{x + 5}{x^2 + 16}\,dx$
15. $\int \frac{e^{-3t}}{e^{-3t} - 3}\,dt$