LESSON 9-3 INTEGRATING TRIGONOMETRIC FUNCTIONS

Six of the integrals for trig functions follow from the rules for derivatives.

Integrals of Trig Functions: (From the derivative rules, where u is a function of x.)

1. $\int \cos x\,dx = \sin x + C$; $\int \cos u\,u'\,dx = \sin u + C$

2. $\int \sin x\,dx = -\cos x + C$; $\int \sin u\,u'\,dx = -\cos u + C$

Be careful with your SIGNS!

3. $\int \sec^2 x\,dx = \tan x + C$; $\int \sec^2 u\,u'\,dx = \tan u + C$

4. $\int \csc^2 x\,dx = -\cot x + C$; $\int \csc^2 u\,u'\,dx = -\cot u + C$

5. $\int \sec x \tan x\,dx = \sec x + C$; $\int \sec u \tan u\,u'\,dx = \sec u + C$

6. $\int \csc x \cot x\,dx = -\csc x + C$; $\int \csc u \cot u\,u'\,dx = -\csc u + C$

Two other rules can be developed by using the identities $\tan x = \dfrac{\sin x}{\cos x}$ and $\cot x = \dfrac{\cos x}{\sin x}$.

Examples: Integrate.

1. $\int \tan x\,dx = \int \dfrac{\sin x}{\cos x}\,dx$

$= -\ln|\cos x| + C$

2. $\int \cot x\,dx = \int \dfrac{\cos x}{\sin x}\,dx$

$= \ln|\sin x| + C$

So,
$\int \tan x\,dx = -\ln|\cos x| + C$; $\int \tan u\,u'dx = -\ln|\cos u| + C$

$\int \cot x\,dx = \ln|\sin x| + C$; $\int \cot u\,u'\,dx = \ln|\sin u| + C$

Examples: Integrate.

3. $\int -3\sin x\,dx = 3\cos x + C$

4. $\frac{1}{\pi}\int \cos(\pi t)\,dt \cdot \pi = \frac{1}{\pi}\sin(\pi t) + C$

5. $\frac{1}{3}\int y^2 \sec^2(y^3)\,dy \cdot 3 = \frac{1}{3}\tan(y^3) + C$

6. $\frac{1}{2}\int \sec(2x+1)\tan(2x+1)\,dx \cdot 2$

$= \frac{1}{2}\sec(2x+1) + C$

7. $\int \dfrac{\cos(5x)}{\sin^2(5x)}\,dx = \frac{1}{5}\int \sin^{-2}(5x)\cos(5x)\cdot \underbrace{5\,dx}_{u'}$

$= -\frac{1}{5}\sin^{-1}(5x) + C$ or $-\frac{1}{5}\csc(5x) + C$

8. $\int \dfrac{\sec^2\theta}{\underbrace{\tan\theta}_{u}}\,d\theta = \ln|\tan\theta| + C$

Example 9: Integrate $\int \tan^2 x \, dx$

Hint: This problem is quite simple to integrate correctly, but it does require a trig substitution. From the Basic Pythagorean Identity, $\sin^2 x + \cos^2 x = 1$, you can get

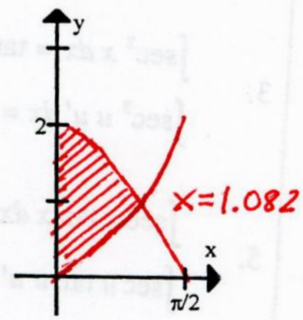
$\tan^2 x + 1 = \sec^2 x$ (by dividing each term by $\cos^2 x$)

or $1 + \cot^2 x = \csc^2 x$ (by dividing each term by $\sin^2 x$)

What substitution can you make to be able to integrate $\int \tan^2 x \, dx$?

$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C$

Example 10: Find the area bounded by $y = 2\cos x$, $y = \frac{1}{2}\tan x$, and $x = 0$. Set up an integral, and use a calculator to integrate.



$A = \int_0^{1.082} (2\cos x - \frac{1}{2}\tan x) \, dx = 1.387$ or 1.388

ASSIGNMENT 9-3

Evaluate the integrals in Problems 1-14.

1. $\int_0^{\frac{\pi}{4}} \sin(2x) \, dx$
2. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 3 \csc t \cot t \, dt$
3. $\int_0^{\frac{2\pi}{3}} \tan\left(\frac{x}{2}\right) dx$
4. $\int (\sec^2 \theta - 2) \, d\theta$
5. $\int \left(\theta^2 + \sec(\theta - 1)\tan(\theta - 1)\right) d\theta$
6. $\int \frac{\cos\left(\frac{1}{x}\right)}{x^2} \, dx$
7. $\int \frac{\cos y}{\sin^3 y} \, dy$
8. $\int \sec^5 x \tan x \, dx$
9. $\int \frac{\csc^2(\pi x)}{\cot(\pi x)} \, dx$
10. $\int \frac{\cos y}{\sin y - 2} \, dy$
11. $\int \frac{\cot^2 x - 1}{\cot x} \, dx$
12. $\int \frac{e^{\tan x - 1}}{\cos^2 x} \, dx$
13. $\int \sin(e^{-t}) e^{-t} \, dt$
14. $\int \tan^2(3x) \, dx$
15. Use a calculator to find the area between the curve $y = |2\cos x + \cos(2x)|$ and the x-axis, from $x = 0$ to $x = \pi$. Show an integral set up and an answer.
16. Use a calculator to find the area of the region in the first quadrant bounded by $y = 3\cos(2x)$, $y = 3x$, and $x = 0$. Show an integral set up and an answer.