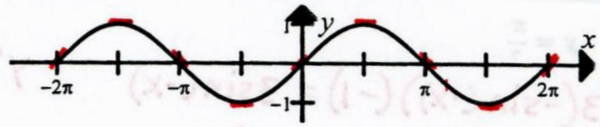
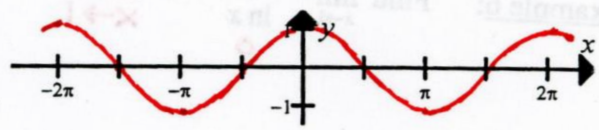
LESSON 9-2 DIFFERENTIATING TRIGONOMETRIC FUNCTIONS

The graph of $f(x) = \sin x$ is shown at right.



Warm-up Example: Estimate slopes for the graph of $f(x) = \sin x$ at $x = -2\pi, \frac{-3\pi}{2}, -\pi, \frac{-\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and 2π.

Plot those slopes in the coordinate plane at right, and connect them to make a smooth continuous curve. This is the graph of $f'(x)$.



$f'(x) =$ $\cos x$ What do you think?

Derivatives of the Trigonometric Functions: u is a function of x.

1. $\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\sin u = \cos u \ u'$
2. $\frac{d}{dx}\cos x = -\sin x$ $\frac{d}{dx}\cos u = -\sin u \ u'$
3. $\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\tan u = \sec^2 u \ u'$
4. $\frac{d}{dx}\cot x = -\csc^2 x$ $\frac{d}{dx}\cot u = -\csc^2 u \ u'$
5. $\frac{d}{dx}\sec x = \sec x \tan x$ $\frac{d}{dx}\sec u = \sec u \tan u \ u'$
6. $\frac{d}{dx}\csc x = -\csc x \cot x$ $\frac{d}{dx}\csc u = -\csc u \cot u \ u'$

Examples: Differentiate.

1. $\frac{d}{dx}\cos(3x^2) = -\sin(3x^2)\cdot 6x$
   $= -6x\sin(3x^2)$

2. $y = \tan^4(5x) = (\tan(5x))^4$
   $y' = 4(\tan(5x))^3 \cdot \sec^2(5x)\cdot 5$

3. $f(t) = \csc\left(\frac{t}{2}\right)$
   $f'(t) = -\csc\left(\frac{t}{2}\right)\cot\left(\frac{t}{2}\right)\cdot\frac{1}{2}$
   $= -\frac{1}{2}\csc\left(\frac{t}{2}\right)\cot\left(\frac{t}{2}\right)$

4. $g(x) = e^{-2x}\cot x$

   Product rule

   $g'(x) = e^{-2x}(-\csc^2 x) + \cot x \cdot e^{-2x}(-2)$

Example 5: Without a calculator, find the slope of the graph of $y = 3\cos(-x)$, where:

a. $x = \frac{\pi}{2}$

$y' = 3(-\sin(-x))(-1) = 3\sin(-x)$

$y'(\frac{\pi}{2}) = 3\sin(-\frac{\pi}{2}) = 3(-1) = -3$

b. $x = \frac{-\pi}{3}$

$y'(-\frac{\pi}{3}) = 3\sin\frac{\pi}{3} = 3(\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}}{2}$

Example 6: Find $\lim_{x\to 1} \frac{\tan(x-1)}{\ln x}$ $\left(\frac{0}{0}\right)$ $= \lim_{x\to 1} \frac{\sec^2(x-1)}{\frac{1}{x}} = \frac{\sec^2 0}{1} = \frac{1}{1} = 1$

Example 7: Find the x-values of the relative extrema for the function $f(x) = \sin x - \cos x$ on $[0, 2\pi]$.

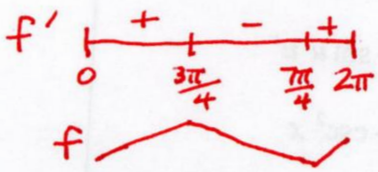
$f'(x) = \cos x - (-\sin x) = \cos x + \sin x$

$f'(x) = 0$ when $\cos x = -\sin x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Rel. max. at $x = \frac{3\pi}{4}$

Rel. min. at $x = \frac{7\pi}{4}$



ASSIGNMENT 9-2

Differentiate in Problems 1-16.

1. $y = 3\cos(t-2)$

2. $f(x) = x - \sin(2x)$

3. $g(x) = \cos^2(x^2+1)$

4. $h(\theta) = \tan(3\theta) - \theta^2$

5. $f(x) = \ln(\cot x)$

6. $y = \csc(2x) - \sec(x^2)$

7. $f(y) = \frac{y^2 - \sec y}{y^3}$

8. $g(t) = t^3 \tan^3(t^3)$

9. $h(x) = \sin(\pi x) + \pi x^2$

10. $f(\theta) = \frac{-2\theta}{\sin\theta}$

11. $y = \csc\frac{1}{x} - 3\sqrt{x}$

12. $P(t) = \sqrt{\cos t - \sin t}$