LESSON 9-1 TRIGONOMETRY WITH A CALCULATOR, GRAPHS OF TRIGONOMETRIC FUNCTIONS

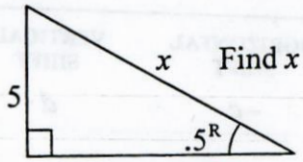
When using a calculator with trig functions, it is important that the calculator is set in the correct mode (radians or degrees). In Calculus, we will deal almost entirely with radian measure. You **will** set your calculator to radian mode prior to taking the AP exam. You will also express all calculator answers to 3 or more decimal place accuracy (unless the problem specifically asks for something else).

Example 1: Use a calculator to find:

a. $\sin 2 = .909$ b. $\tan\left(\frac{-\pi}{5}\right) = -.726$ (or $-.727$) c. $\sec 1.3 = 3.738$

Example 2: Use a calculator to find the missing measure in each triangle.

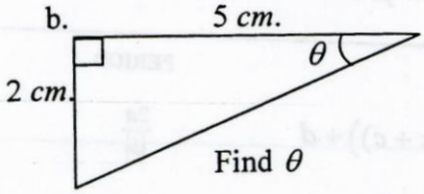
a.



$\sin .5 = \frac{5}{x}$

$x = \frac{5}{\sin .5} = 10.429$

b.



$\tan\theta = \frac{2}{5}$

$\theta = \tan^{-1}\frac{2}{5} = .380$ or $.381$

Graphs of Trig Functions:

Coterminal angles are angles having the same terminal side if placed in standard position. The fact that coterminal angles have the same trig ratios should lead you to believe that the graphs of trig functions would "repeat" every 2π radians (measured on the x-axis). They do. Actually, tangent and cotangent graphs repeat more often (every π radians).

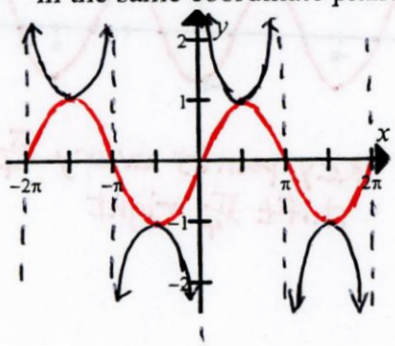
Trig functions are periodic (their graphs repeat after a certain period or cycle). The sine, cosine, cosecant, and secant functions all have a period of 2π. The tangent and cotangent functions have a period of π.

You should be able to easily graph the trig functions by using trig values at $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, and by using the fact that the functions are periodic.

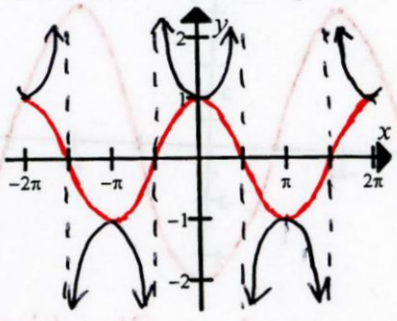
(You should also use $x = \pm\frac{\pi}{4}$ for the tangent and cotangent graphs.)

Example 3: Graph each of the following.

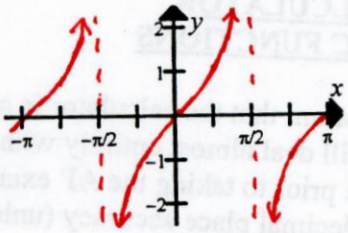
a. $y = \sin x$ and $y = \csc x$ in the same coordinate plane



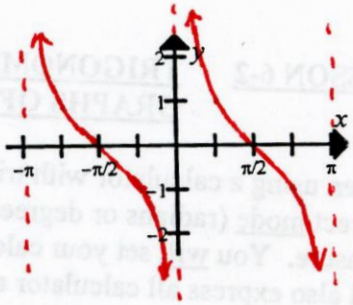
b. $y = \cos x$ and $y = \sec x$ in the same coordinate plane



c. $y = \tan x$



d. $y = \cot x$



Remember: Each of these 2 functions has a period of π.

You should be able to use the parent trig graphs to graph functions of the form $y = a\sin\big(b(x+c)\big)+d$. (sin could be replaced by any other trig function.)

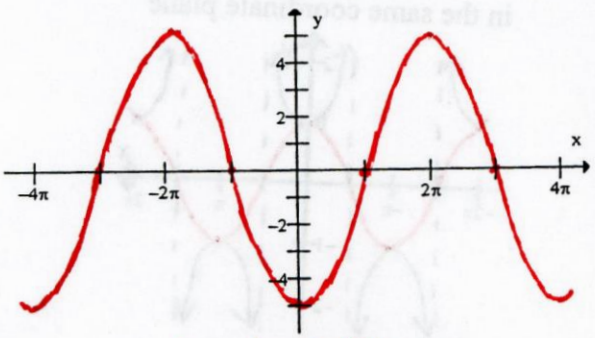
The chart below provides an aid, but remember to think of "adjustments to graphs."

FUNCTION	PERIOD	AMPLITUDE	HORIZONTAL SHIFT	VERTICAL SHIFT
$y = a\sin\big(b(x+c)\big)+d$ or $y = a\cos\big(b(x+c)\big)+d$	$\frac{2\pi}{\lvert b\rvert}$	$\lvert a\rvert$	$-c$	d
$y = a\tan\big(b(x+c)\big)+d$ or $y = a\cot\big(b(x+c)\big)+d$	$\frac{\pi}{\lvert b\rvert}$	None	$-c$	d
$y = a\sec\big(b(x+c)\big)+d$ or $y = a\csc\big(b(x+c)\big)+d$	$\frac{2\pi}{\lvert b\rvert}$	None	$-c$	d

When c is positive, the horizontal shift is to the left. When c is negative, the horizontal shift is to the right. Horizontal shift is often called phase shift for periodic functions.

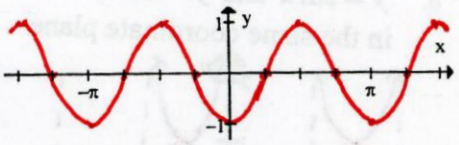
Example 4: Without using a calculator, sketch two cycles of:

a. $f(x) = -5\cos\left(\frac{x}{2}\right)$ Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$ Amp: 5



x-axis reflection
no shifts

b. $g(t) = \sin\left(2t - \frac{\pi}{2}\right) = \sin\left(2\left(t - \frac{\pi}{4}\right)\right)$ Period $= \frac{2\pi}{2} = \pi$ Amp = 1



Key points every $\frac{\pi}{4}$
shift $\frac{\pi}{4}$ right

The sine and cosine functions are related to each other by the basic Pythagorean Identity:

$$\begin{cases} \sin^2\theta + \cos^2\theta = 1 \\ \text{or } \sin^2 x + \cos^2 x = 1 \end{cases}$$

Example 5: Use the Pythagorean Identity to rewrite $2\cos\theta - \sin^2\theta = -2$ in a form which only contains one trig function. Then, without using a calculator, solve for θ on the interval $[0, 2\pi)$.

$2\cos\theta - (1 - \cos^2\theta) = -2$

$2\cos\theta - 1 + \cos^2\theta = -2$

$\cos^2\theta + 2\cos\theta + 1 = 0$

$(\cos\theta + 1)^2 = 0$

$\cos\theta = -1$

$\theta = \pi$

Use your calculator to verify your solution.

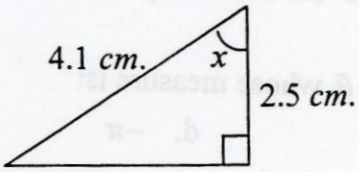
ASSIGNMENT 9-1

1. Use a calculator to find (to 3 or more decimal place accuracy):

   a. $\cos\frac{3\pi}{5}$ b. $\csc(-2.9)$ c. $\tan\left(\frac{5}{8}\right)$ d. $\cot(-1.6\pi)$

For Problems 2-5, use a calculator to find the values of x in each triangle.

2.



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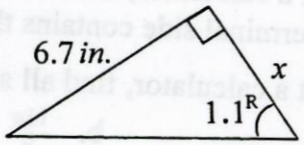

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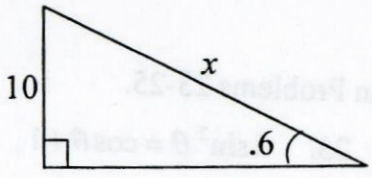
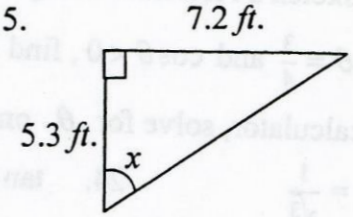


5.



Use a calculator to solve for x on the interval $[0, 2\pi)$ for Problems 6 and 7.

6. $\sin x - 2\cos x = 0$

7. $\tan x = \csc^2 x - 2$