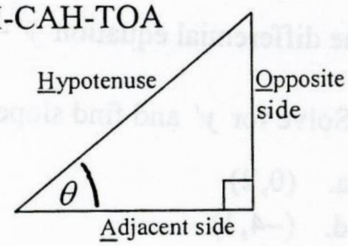
# LESSON 8-5 REVIEW OF BASIC TRIGONOMETRY

**Basic Right Triangle Trigonometry:**

The basic right triangle trigonometric ratios are given by SOH-CAH-TOA




$\underline{\text{s}}\text{ine}\,\theta = \dfrac{opp}{hyp}$ (SOH) $\qquad$ cosecant $\theta = \dfrac{1}{\sin\theta}$

$\underline{\text{c}}\text{osine}\,\theta = \dfrac{adj}{hyp}$ (CAH) $\qquad$ secant $\theta = \dfrac{1}{\cos\theta}$

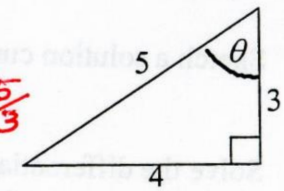
$\underline{\text{t}}\text{angent}\,\theta = \dfrac{opp}{adj}$ (TOA) $\qquad$ cotangent $\theta = \dfrac{1}{\tan\theta}$

(Reciprocal Functions)

$0^\circ < \theta < 90^\circ$

When using right triangle trigonometry, angles are usually measured in degrees.

Example 1: Use the triangle at right to find

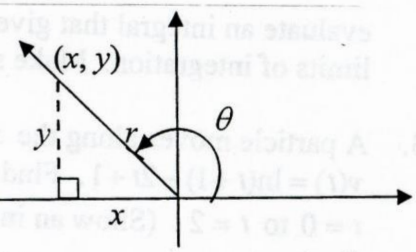



a. $\sin\theta = \frac{4}{5}$ $\quad$ b. $\cos\theta = \frac{3}{5}$ $\quad$ c. $\tan\theta = \frac{4}{3}$ $\quad$ d. $\sec\theta = \frac{5}{3}$

**Trigonometric Functions Defined as Circular Functions:**

Angles in a right triangle must be positive and less than or equal to 90°. A less restrictive way of defining trigonometric (trig) ratios is to use angles which can be any measure.

At right is an angle in standard position. The vertex of the angle is the origin. The initial side of the angle is the positive $x$-axis. In the figure shown, the terminal side was formed by a counter-clockwise rotation, so the measure of the angle, $(\theta)$, is positive. Clockwise rotations produce negative angles.




When trig functions are defined using rotations from an initial ray (side) in the coordinate plane, they are called circular functions. In Calculus, angles are usually defined by circular trig functions and are almost always measured in radians. $(2\pi^{R} = 360^\circ)$

The circular function trig definitions are (see figure):

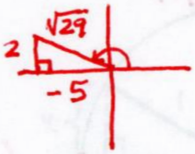
$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\qquad$ $\cot\theta = \dfrac{x}{y}$

- $\theta$ is any measure
- $r = \sqrt{x^2 + y^2}$ (Positive)
- $x$ and $y$ may be +, −, or 0

Example 2: Find $\sin\theta$, $\csc\theta$, and $\cot\theta$, if $\theta$ is an angle in standard position whose terminal side passes through the point $(-5,2)$.

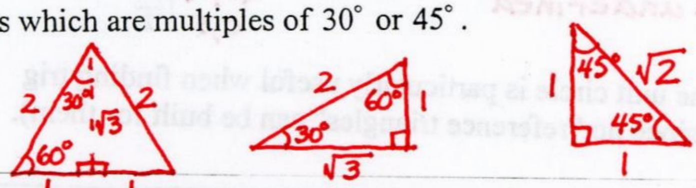



$r=\sqrt{25+4}=\sqrt{29}$ $\quad \sin\theta = \dfrac{2}{\sqrt{29}}$ $\quad \cot\theta = -\dfrac{5}{2}$

$\csc\theta = \dfrac{\sqrt{29}}{2}$

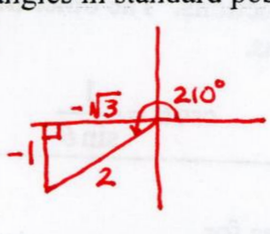
Circular function trigonometry makes use of reference angles in triangles and is really not much different than right triangle trigonometry. Think of it as an extension of right triangle trig.

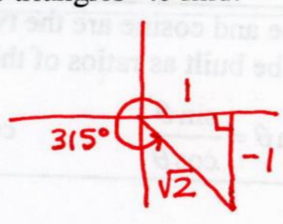
$30^\circ$ - $60^\circ$ - $90^\circ$ and $45^\circ$ - $45^\circ$ - $90^\circ$ reference triangles can be used to find trig ratios of angles which are multiples of $30^\circ$ or $45^\circ$.




Example 3: Draw angles in standard position and make "reference triangles" to find:

a. $\cos 210^\circ = -\dfrac{\sqrt{3}}{2}$ $\qquad$ b. $\tan 315^\circ = -1$




Example 4: Since $2\pi$ radians $= 360^\circ$, it follows that $\pi^R = 180^\circ$, and the following common radian measures should be easy to think about in degrees. Convert each common radian measure to degrees.

a. $\frac{\pi}{2} = 90^\circ$ $\qquad$ b. $\frac{\pi}{4} = 45^\circ$ $\qquad$ c. $\frac{\pi}{3} = 60^\circ$ $\qquad$ d. $\frac{\pi}{6} = 30^\circ$

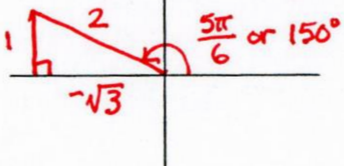
Example 5: Convert from radians to degrees or degrees to radians without using a calculator.

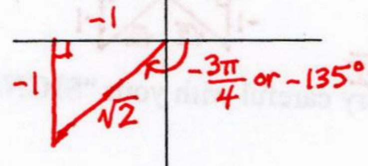
a. $\frac{5\pi}{4} = 5\left(\frac{\pi}{4}\right) = 5(45^\circ) = 225^\circ$ $\qquad$ b. $270^\circ = 3(90^\circ) = \frac{3\pi}{2}$ $\qquad$ c. $-120^\circ = -2(60^\circ) = -\frac{2\pi}{3}$

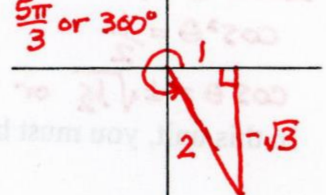
or $\frac{5\pi}{4}\cdot\frac{180^\circ}{\pi} = 225^\circ$ $\qquad$ or $270^\circ \cdot \frac{\pi}{180^\circ} = \frac{3\pi}{2}$ $\qquad$ or $-120^\circ \cdot \frac{\pi}{180^\circ} = -\frac{2\pi}{3}$

Examples: Draw angles in standard position, and make "reference triangles" to find the following without using a calculator:

6. $\tan\frac{5\pi}{6} = \frac{1}{-\sqrt{3}}$ $\qquad$ 7. $\cos\left(\frac{-3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$ $\qquad$ 8. $\csc\frac{5\pi}{3} = \frac{2}{-\sqrt{3}}$

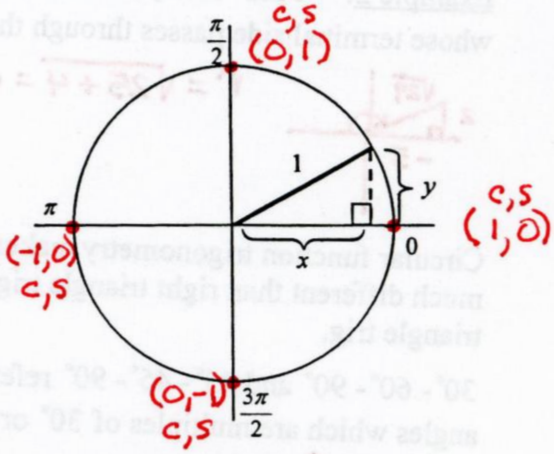


A unit circle is created by letting $r = 1$ when dealing with the circular trig functions.

Then, $\sin\theta = y$, $\cos\theta = x$, and $\tan\theta = \dfrac{y}{x}$.




Example 9: Use a unit circle to find:

a. $\sin\frac{\pi}{6} = \frac{1}{2}$ b. $\sin 0 = 0$ c. $\cos 0 = 1$

d. $\sin\frac{\pi}{2} = 1$ e. $\cos\frac{\pi}{2} = 0$ f. $\sin\pi = 0$

g. $\tan\pi = 0$ h. $\sin\frac{3\pi}{2} = -1$ i. $\cos\frac{3\pi}{2} = 0$

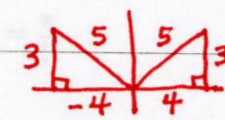
j. $\cos(-\pi) = -1$ k. $\tan\left(\frac{-\pi}{2}\right)$ = undefined

As you can see from Example 9, the unit circle is particularly useful when finding trig ratios for the quadrant separators (since no "reference triangles" can be built for them).

| |
|---|
| Sine and cosine are the two most important trig functions. The other trig functions can all be built as ratios of the sine and cosine functions. |
| $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ |

Example 10: If $\sin\theta = \frac{3}{5}$, find the possible values for

a. $\csc\theta = \frac{5}{3}$ b. $\cos\theta = \pm\frac{4}{5}$ c. $\tan\theta = \pm\frac{3}{4}$




Solving trigonometric equations requires you to "**work backwards**" from ratios to angles.

Example 11: Solve the following trig equations without using a calculator. Find all of the solutions in the interval $[0, 2\pi)$.

a. $\csc x = \frac{-2}{\sqrt{3}}$


$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

b. $\cot\theta = \sqrt{3}$


$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

c. $2\cos^2\theta - 1 = 0$ **Note**: $\cos^2\theta$ means $(\cos\theta)^2$. This is trig symbolism.

$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\sqrt{\frac{1}{2}}$ or $\pm\frac{1}{\sqrt{2}}$


$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

In this unit, you must be very careful with your "**SIGNS.**"