LESSON 8-4 L' HOPITAL'S RULE

Some limits cannot be found using algebraic methods. If direct substitution produces one of these two indeterminate forms $\left(\frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty}\right)$, then a rule known as **L' Hopital's Rule** may help you find the limit.

L' Hopital's Rule

If $\lim\limits_{x\to\square}\frac{f(x)}{g(x)}=\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then $\lim\limits_{x\to\square}\frac{f(x)}{g(x)}=\lim\limits_{x\to\square}\frac{f'(x)}{g'(x)}$

To use L' Hopital's Rule, you must have the limit of an expression which is written in fractional form.

Examples: Evaluate.

1. $\lim\limits_{x\to\infty}\frac{\overset{\infty}{x}}{\underset{\infty}{e^x}} = \lim\limits_{x\to\infty}\frac{1}{e^x} = 0$

2. $\lim\limits_{x\to\infty}\frac{\overset{\infty}{e^x}}{\underset{\infty}{x}} = \lim\limits_{x\to\infty}\frac{e^x}{1} = \infty$ or DNE

3. $\lim\limits_{x\to 0}\frac{\overset{0}{x}}{\underset{1}{e^x}} = 0$

4. $\lim\limits_{x\to 0^-}\frac{\overset{1}{e^x}}{\underset{0^-}{x}} = -\infty$ or DNE

5. $\lim\limits_{x\to 0}\frac{\overset{0}{3-3e^{3x}}}{\underset{0}{x}} = \lim\limits_{x\to 0}\frac{-9e^{3x}}{1} = -9$

Do not use L' Hopital's Rule just because a problem "looks like" a candidate for the rule. Examples 3 and 4 are not candidates for L' Hopital's Rule, because direct substitution does not produce $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$.

If using L' Hopital's Rule leaves you with the form $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then you can use L' Hopital's Rule again. It is a process which can be repeated as many times as necessary. Just remember to use direct substitution at each step to make sure L' Hopital's Rule can be used (check for $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ form).

Examples: Evaluate.

6. $\lim\limits_{x\to-\infty}\frac{\overset{\infty}{x^2}}{\underset{\infty}{e^{-x}}} = \lim\limits_{x\to-\infty}\frac{\overset{-\infty}{2x}}{\underset{-\infty}{-e^{-x}}} = \lim\limits_{x\to-\infty}\frac{2}{e^{-x}}$

$= \frac{2}{\infty} = 0$

7. $\lim\limits_{x\to 1}\frac{\overset{0}{\ln x - x + 1}}{\underset{0}{x^2-2x+1}} = \lim\limits_{x\to 1}\frac{\overset{0}{\frac{1}{x}-1}}{\underset{0}{2x-2}}$

$= \lim\limits_{x\to 1}\frac{\frac{-1}{x^2}}{2} = -\frac{1}{2}$