LESSON 8-3 SLOPE FIELDS

A **slope field** is a graphical representation of a set of slopes obtained from a differential equation. Remember that a differential equation involves a derivative. That derivative represents the slopes for a function. In Lesson 5-7, you learned to solve differential equations by separating variables. Even if you cannot separate variables and integrate, you can still use a differential equation to plot the slopes for a function.

Example 1: Find the slopes given by the differential equation $\frac{dy}{dx} = \frac{-x}{y}$ at the following points:

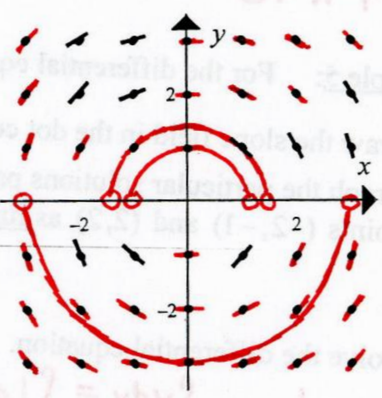
a. $(3, 2)$ b. $(-1, 3)$ c. $(-2, -1)$ d. $(2, -2)$

$-\frac{3}{2}$ $\frac{1}{3}$ -2 1

Why can't you find slopes when $y = 0$? Division by 0 is illegal. slopes are undefined

Example 2: Find and plot the slopes given by $\frac{dy}{dx} = \frac{-x}{y}$ for each remaining marked point (dot) in the coordinate plane at right.

Hint: There are 24 more slopes to plot (places where the slope exists).



Example 3: In Example 2, you made what is known as a slope field. Starting at the point $(0, 1)$, follow the flow of the slopes to sketch the solution curve containing $(0, 1)$. Your graph should be "parallel" to the slope lines and be like an "average of slopes" whenever it goes between lines. Your solution curve must represent a function whose domain is the largest possible open interval containing the given point. Sketch a solution curve passing through $(-1, 1)$ and one passing through $(0, -3)$. What type of graph does this differential equation seem to be producing? semi-circles

Note: The most common student error in sketching a particular solution to a differential equation is to extend the sketch too far and create a graph which is not a function. It is important to set appropriate "boundaries" for your sketch. Why is the x-axis a "boundary" for the differential equation from Examples 1 and 2?

Example 4: Solve the differential equation $\frac{dy}{dx} = \frac{-x}{y}$.

$\int y\,dy = \int -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$ or $x^2 + y^2 = C$ (circle)

$y^2 = -x^2 + C$

Note: Solving for y yields $y = \sqrt{-x^2+C}$ or $y = -\sqrt{-x^2+C}$.

Do the possible solutions verify your results from Example 3?

Remember that the solution to a differential equation is a function. The particular solution for the differential equation $\frac{dy}{dx} = \frac{-x}{y}$ whose graph passes through the point $(0,1)$ is

$y = \sqrt{-x^2+C} \rightarrow 1 = \sqrt{0+C} \quad C = 1 \quad y = \sqrt{-x^2+1}$

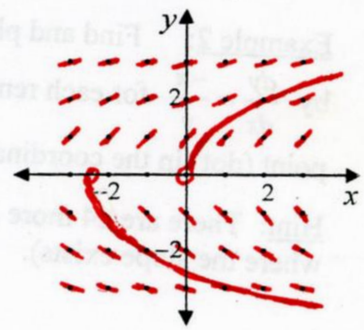
The particular solution whose graph passes through the point $(0,-3)$ is

$y = -\sqrt{-x^2+C} \rightarrow -3 = -\sqrt{0+C} \quad 9 = C \quad y = -\sqrt{-x^2+9}$

Example 5: For the differential equation $y' = \frac{1}{y}$

a. Draw the slope field in the dot coordinate plane at right.

b. Graph the particular solutions passing through the points $(-2,-1)$ and $(2,2)$ as functions of x.



c. Solve the differential equation.

$\frac{dy}{dx} = \frac{1}{y} \quad \int y\,dy = \int 1\,dx \quad y^2 = 2x + C$

$\frac{1}{2}y^2 = x + C_1 \quad y = \sqrt{2x+C}$ or $y = -\sqrt{2x+C}$

d. Write as functions the particular solutions for the differential equation whose graphs pass through $(-2,-1)$ and $(2,2)$.

For $(-2,-1)$: $-1 = -\sqrt{-4+C}$

$1 = -4 + C$

$5 = C \quad y = -\sqrt{2x+5}$

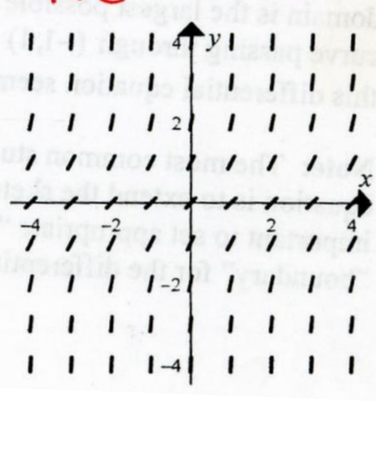
For $(2,2)$: $2 = \sqrt{4+C}$

$4 = 4 + C$

$0 = C$

$y = \sqrt{2x}$

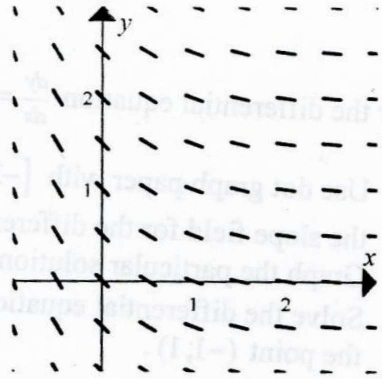
Example 6: Which of the differential equations below matches the slope field shown at right?



a. $y' = x$ b. $y' = y$ c. $y' = x - y$

d. $y' = 1 + y^2$ e. $y' = 1 + x^2$

Example 7: The slope field for a certain differential equation is shown at the right. Which of the following could be a specific solution to the differential equation?



a. $y = e^x$ b. $y = e^{-x}$ c. $y = -e^x$

d. $y = -\ln x$ e. $y = \ln x$

Using Lessons 5-9 and 5-7, you should be able to show graphically a particular solution of a differential equation, and confirm that solution by solving the differential equation (if it is possible to do so).

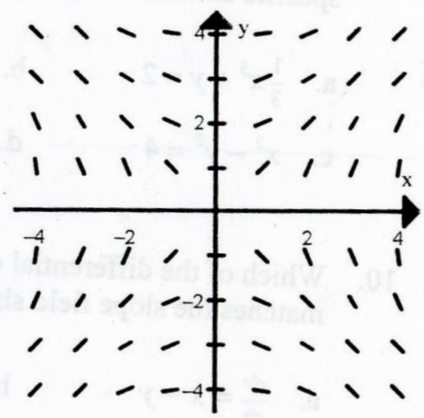
ASSIGNMENT 8-3

1. Find the slopes given by the differential equation $y' = \dfrac{x^2}{y-2}$ at each of the following points:

   a. $(0,0)$ b. $(1,1)$ c. $(-2,4)$ d. $(4,-2)$ e. $(-3,-3)$ f. $(5,12)$

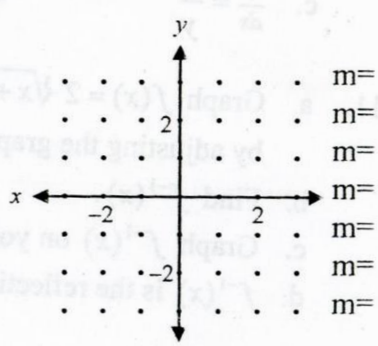
2. For the differential equation in Problem 1, why are there no slopes when $y = 2$?

3. The slope field for $y' = \dfrac{x}{y}$ is shown at right.



   a. Plot the following points on the slope field:
      i. $(1,2)$ ii. $(3,1)$ iii. $(0,3)$
      iv. $(0,-2)$ v. $(-2,-1)$
   b. Plot a solution curve through each of the points from Part a. Remember that the curves have to be functions.
   c. What would a solution curve containing (2, 2) look like?
   d. Solve the differential equation $y' = \dfrac{x}{y}$.

4. For the differential equation $\dfrac{dy}{dx} = y$



   a. Use dot paper to draw the slope field for the differential equation.
   b. Graph the particular solutions passing through the points $(0,1)$ and $(0,-1)$.
   c. Solve the differential equation, and find the particular solutions that contain the points $(0, 1)$ and $(0, -1)$.