Lesson 8-2 Integral Test and p-Series

Integral Test:
If the function $f(x)$ is <u>positive</u>, <u>continuous</u>, and <u>decreasing</u> for $x \geq 1$ and $a_n = f(n)$ then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

Note: $\sum_{n=1}^{\infty} a_n \neq \int_1^{\infty} f(x)\,dx$

Examples: Use the integral test to determine convergence or divergence.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

$\lim_{b\to\infty} \int_1^b \frac{1}{x^2+1}\,dx$

$\lim_{b\to\infty} \arctan x \Big|_1^b$

$\lim_{b\to\infty} \arctan b - \arctan 1$

$\frac{\pi}{2} - \frac{\pi}{4}$

$\frac{\pi}{4}$

The series converges by IT

2. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$\lim_{b\to\infty} \int_2^b \frac{x^{-1}}{\ln x}\,dx$

$\lim_{b\to\infty} \ln|\ln x| \Big|_2^b$

$\lim_{b\to\infty} \ln|\ln b| - \ln|\ln 2|$

diverges

The series diverges by IT.

Examples: Use the integral test to determine convergence or divergence of these series.

3. $\sum_{n=1}^{\infty} \frac{1}{n}$

$\lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx$

$\lim_{b\to\infty} \ln|x| \Big|_1^b$

$\lim_{b\to\infty} \ln|b| - \ln 1$

diverges

The series also div. by I.T.

4. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$\lim_{b\to\infty} \int_1^b \frac{1}{\sqrt{x}}\,dx$

$\lim_{b\to\infty} \int_1^b x^{-\frac{1}{2}}\,dx$

$\lim_{b\to\infty} 2x^{\frac{1}{2}} \Big|_1^b$

$\lim_{b\to\infty} 2\sqrt{b} - 2$

div.

The series div. by IT.

5. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\lim_{b\to\infty} \int_1^b x^{-2}\,dx$

$\lim_{b\to\infty} -x^{-1} \Big|_1^b$

$\lim_{b\to\infty} -\frac{1}{b} + 1$

1

The series conv by I.T.

p-Series and Harmonic Series:

If p is a <u>positive</u> <u>constant</u> then $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ is called a **p-series.**

The last three examples are all p-series. Each of them could have been done using the following test.

> **p-series Test**
> If $p > 1$ then the p-series **converges.** If $0 < p \le 1$ then the p-series **diverges.**

The **harmonic series** is the p-series in which $p = 1$. $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$ (Example 3 above)

Examples: Use the p-series test to determine convergence or divergence of these series.

6. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$p = \frac{3}{2} > 1$ conv. by p-s Test

7. $\sum_{n=1}^{\infty} n^3 \sqrt[3]{n^{-11}} = \sum_{n=1}^{\infty} n^3 \cdot n^{-\frac{11}{3}}$

$= \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$p = \frac{2}{3} < 1$

div. by p-s T