LESSON 8-1 DIFFERENTIAL EQUATIONS

Differential Equations are equations with derivatives in them. In this course, you will only learn how to solve the simplest type of differential equations, in which you can separate variables. You may be asked to find a general solution of the differential equation (which gives you a family of curves) or a particular solution (which gives you a single curve).

Procedure for Solving Differential Equations

1. Rewrite y' as $\frac{dy}{dx}$ (if necessary).
2. Multiply both sides of the equation by dx (if necessary).
3. Separate variables. (This is the most crucial step.)
4. Integrate both sides of the equation. (Remember to add C to one side.)
5. Solve for y (if necessary).
6. Use an initial condition to solve for C (if an initial condition is given).

Steps 5 and 6 are interchangeable.

Example 1: Find a general solution of $x+2yy'=0$

First, rewrite as $x+2y\frac{dy}{dx}=0$. Then, $x\,dx+2y\,dy=0$

$\int 2y\,dy=\int -x\,dx$

$y^2=-\frac{1}{2}x^2+C$

Write your solution to Example 1 as a pair of possible functions (in the form $y=f(x)$) for the particular solutions to the differential equation. $y=\sqrt{-\frac{1}{2}x^2+C}$ or $y=-\sqrt{-\frac{1}{2}x^2+C}$

Example 2: Find an equation of a function which contains the point $(0,-3)$, and whose slope is $\frac{xe^{x^2}}{y}$ for each point (x, y) on the curve.

$\frac{dy}{dx}=\frac{xe^{x^2}}{y}$

$\int y\,dy=\frac{1}{2}\int xe^{x^2}dx\,(2)$

$\frac{1}{2}y^2=\frac{1}{2}e^{x^2}+C_1$

$y^2=e^{x^2}+C$

$y=-\sqrt{e^{x^2}+C}$

Why the negative root?

$-3=-\sqrt{e^0+C}$

$+3=+\sqrt{1+C}$

$9=1+C$

$8=C$

$y=-\sqrt{e^{x^2}+8}$

If $|y|=2$, then $y=\pm 2$
Since $e^a \cdot e^b = e^{a+b}$, $e^{a+b} = e^a \cdot e^b$

Example 3:

a. Find a general solution of $y-2 = x\frac{dy}{dx}$.

$(y-2)\,dx = x\,dy$

$\int \frac{1}{x}dx = \int \frac{1}{y-2}dy$

$e^{\ln|x|+C_1} = e^{\ln|y-2|}$

$e^{\ln|x|} \cdot e^{C_1} = e^{\ln|y-2|}$

$|x| \cdot e^{C_1} = |y-2|$

$\pm e^{C_1} \cdot x = y-2$

$Cx = y-2$

$y = Cx+2$

b. Find a particular solution of $y-2 = x\frac{dy}{dx}$ if $y(1)=\frac{1}{2}$.

$\frac{1}{2} = C(1)+2 \quad -\frac{3}{2} = C \quad y = -\frac{3}{2}x+2$

ASSIGNMENT 8-1

For Problems 1-4, find a general solution of each differential equation. *Solve for y in Problems 2 and 4.

1. $y' = \dfrac{x^2-1}{2y^2+3}$ *2. $e^x y\frac{dy}{dx} = 1$ 3. $2xy' = y+1$ *4. $(x-2)\frac{dy}{dx} = 2y$

For Problems 5-7, find a particular solution of the differential equation with the given initial condition. (Remember to write your solutions in the form $y = f(x)$.)

5. $\dfrac{dy}{dx} = \dfrac{-2x}{y}$ and $y(2) = -4$ 6. $y = -3x\frac{dy}{dx}$ and $y(1) = e$

7. $\frac{dy}{dt} = ky$ (where k is some constant), and $y(0) = 100$

For Problem 7, write y as a function of k and t.

8. Find an equation of a function which contains the point $(-2,1)$ and whose slope is $\dfrac{x}{2y}$ for each point on the graph of the function.

Simplify in Problems 9 and 10.

9. $\ln\dfrac{e^2}{e^{\sqrt{x}}}$ 10. $e^{\ln a - \ln b}$

Differentiate in Problems 11-13.

11. $y = 3^{2t-1}t^2$ 12. $f(y) = \dfrac{e^{\sqrt{y}}}{y^2}$ 13. $f(x) = e^x \ln x$