LESSON 7-6 INTEGRATION INVOLVING THE NATURAL LOG FUNCTION

Differentiation and integration are inverse operations.

So if $\frac{d}{dx}\ln|x| = \frac{1}{x}$, then $\int \frac{1}{x}\,dx = \ln|x| + C$, and if $\frac{d}{dx}\ln|u| = \frac{u'}{u}$, then $\int \frac{u'}{u}\,dx = \ln|u| + C$.

Log Rules:	$\int \frac{1}{x}\,dx = \ln|x| + C$	and	$\int \frac{u'}{u}\,dx = \ln|u| + C$

Note: Although it is true that both $\frac{d}{dx}\ln x = \frac{1}{x}$ and $\frac{d}{dx}\ln|x| = \frac{1}{x}$, $\int \frac{1}{x}\,dx = \ln|x|$ only. **Why?** Protection of domain

Examples: Integrate

Why doesn't this work as a power function?

1. $\int \frac{-3}{x}\,dx = -3\int \frac{1}{x}\,dx$

$= -3\ln|x| + C$

2. $\frac{1}{2}\int \frac{P}{P^2+1}\,dP \cdot 2$ (u', u)

$= \frac{1}{2}\ln|P^2+1| + C$

or $\frac{1}{2}\ln(P^2+1) + C$

3. $\int \frac{9t^2-6t}{t^3-t^2}\,dt = 3\int \frac{3t^2-2t}{t^3-t^2}\,dt$ ($u' = 3t^2-2t$, u)

$= 3\ln|t^3-t^2| + C$

When integrating a fraction where the degree of the numerator ≥ the degree of the denominator, you will have to use long division (or creative thinking) to "split the fraction."

Example 4: Integrate $\int \frac{x^2-4x+2}{x^2+2}\,dx$

$$\begin{array}{r} 1 + \frac{-4x}{x^2+2} \\ x^2+2 \enclose{longdiv}{x^2 - 4x + 2} \\ \underline{x^2 \qquad\;\; +2} \\ -4x \end{array}$$

$= \int 1\,dx - \int \frac{4x}{x^2+2}\,dx$

$u = x^2+2$

$u' = 2x$

$= \int 1\,dx - 2\int \frac{2x}{x^2+2}\,dx = x - 2\ln(x^2+2) + C$

When integrating functions that contain logarithms, you usually want to think about doing a "Reverse Chain" integration, where $u = \ln x$ (or something containing $\ln x$) and $u' = \frac{1}{x}$ (or something similar).

Example 5: Integrate $\int \frac{\ln x}{x}\,dx$

Hint: $\int \frac{\ln x}{x}\,dx = \int (\ln x)\frac{1}{x}\,dx$

$= \int (\ln x)^1 \cdot \frac{1}{x}\,dx$

$= \frac{1}{2}(\ln x)^2 + C$

Example 6: Integrate $\int \frac{1}{x(2-\ln x)^3}\,dx$

$= -\int (2-\ln x)^{-3}\left(-\frac{1}{x}\right)dx$

$= -\frac{1}{-2}(2-\ln x)^{-2} + C$

$= \frac{1}{2}(2-\ln x)^{-2} + C$

Example 7: Integrate $\int \frac{1}{\sqrt{x}-1}\,dx = \int \frac{1}{u-1}\cdot 2u\,du = \int \frac{2u}{u-1}\,du$

$u = \sqrt{x}$
$u^2 = x$
$2u\,du = dx$

$$\begin{array}{r} 2 + \frac{2}{u-1} \\ u-1 \overline{) \; 2u \quad} \\ \underline{-2u+2} \\ 2 \end{array}$$

$= \int 2\,du + 2\int \frac{1}{u-1}\,du$

$= 2u + 2\ln|u-1| + C$

$= 2\sqrt{x} + 2\ln|\sqrt{x}-1| + C$

ASSIGNMENT 7-6

Evaluate each integral in Problems 1-14.

1. $\int_1^e \frac{2}{x}\,dx$
2. $\int_1^3 \frac{4}{2t-1}\,dt$
3. $\int_{-1}^1 4^{x+1}\,dx$
4. $\int_0^2 e^{-2x}\,dx$
5. $\int e^{x^2-1}x\,dx$
6. $\int \frac{x}{x^2-1}\,dx$
7. $\int \frac{x^2-1}{x}\,dx$
8. $\int \frac{4y-6}{y^2-3y+2}\,dy$
9. $\int \frac{x+1}{x-1}\,dx$
10. $\int \frac{-1}{(x+1)^3}\,dx$
11. $\int \frac{3u}{\sqrt[3]{u^2+1}}\,du$
12. $\int \frac{\sqrt{\ln x}}{x}\,dx$
13. $\int \frac{2}{x(1+\ln x)^5}\,dx$
14. $\int \frac{4}{2+\sqrt{x}}\,dx$

For Problems 15 and 16, find the inverse of the given function, and then sketch the function and its inverse in the same coordinate plane.

15. $f(x) = \ln(-x)$
16. $g(x) = e^{2x} + 1$

Simplify the expressions in Problems 17 and 18 without using a calculator.

17. $\log_3 \frac{1}{27}$
18. $\ln \frac{e^{10}}{e^3}$

19. Use Log Properties to expand $\ln\left(x\sqrt{y-1}\right)$.