LESSON 7-5 DIFFERENTIATING LOGARITHMIC FUNCTIONS

We can find the derivative of the natural log function by using the formula for the Derivative of Inverse Functions from Lesson 5-3 (page 137).

$$g'(x) = \frac{1}{f'(g(x))} \quad \text{where } f \text{ and } g \text{ are inverse functions.}$$

Since $y = e^x$ and $y = \ln x$ are inverse functions, let $f(x) = e^x$ and $g(x) = \ln x$.

Then, $f'(x) = e^x$, so $\frac{d}{dx}\ln x = g'(x) = \frac{1}{f'(\ln x)} = \frac{1}{e^{\ln x}} = \frac{1}{x}$.

Differentiating Logarithmic Functions:

$\frac{d}{dx}\ln x = \frac{1}{x}$ $\qquad$ $\frac{d}{dx}\ln u = \frac{1}{u}u' = \frac{u'}{u}$ (Chain rule form)

$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$ $\qquad$ $\frac{d}{dx}\log_a u = \frac{u'}{u\ln a}$ (Chain rule form)

Examples: Differentiate.

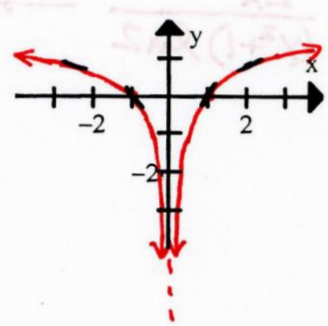
Product rule

1. $y = \ln(5x)$ $\qquad$ 2. $f(t) = \ln(3t^2 - t)$ $\qquad$ 3. $h(x) = x\ln x$

$y' = \frac{5}{5x} = \frac{1}{x}$ $\qquad$ $f'(t) = \frac{6t-1}{3t^2-t}$ $\qquad$ $h'(x) = x\cdot\frac{1}{x} + \ln x\cdot 1$

Example 4:

a. Graph $y = \ln|x|$ in the coordinate plane at right.



b. If $x > 0$, $\ln|x| = \ln x$, so $\frac{d}{dx}\ln|x| = \frac{1}{x}$ for $x > 0$.

What is $\frac{d}{dx}\ln|x|$ when $x < 0$?

Plot some slopes and think about it.

$\frac{d}{dx}\ln|x| = \frac{1}{x}$ for $x < 0$.

When differentiating the natural log function, $\frac{d}{dx}\ln|x| = \frac{d}{dx}\ln x$ and $\frac{d}{dx}\ln|u| = \frac{d}{dx}\ln u$.

In these cases, absolute value may change the domain of the function – but not the derivative.

Example 5:

Find $\frac{d}{dy}\ln\left|5-2y^3\right| = \dfrac{-6y^2}{5-2y^3}$

When possible, simplify logarithmic functions before differentiating them.

Example 6: Differentiate $y = \ln\dfrac{x\sqrt{2x+1}}{x^2+1}$.

First, rewrite as $y = \ln x + \frac{1}{2}\ln(2x+1) - \ln(x^2+1)$

Then, $y' = \dfrac{1}{x} + \dfrac{1}{2}\cdot\dfrac{2}{2x+1} - \dfrac{2x}{x^2+1} = \dfrac{1}{x} + \dfrac{1}{2x+1} - \dfrac{2x}{x^2+1}$

Example 7: Use the Change of Base Formula to show that $\frac{d}{dx}\log_a x = \dfrac{1}{x\ln a}$.

(See page 146 – Differentiating Logarithmic Functions)

$$\frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{d}{dx}\left(\frac{1}{\ln a}\cdot\ln x\right) = \frac{1}{\ln a}\cdot\frac{1}{x} = \frac{1}{x\ln a}$$

Example 8: If $y = \log_2\left(x^2+1\right)$, find $y'(2)$

$$y' = \frac{2x}{(x^2+1)\ln 2} \longrightarrow y'(2) = \frac{4}{5\ln 2}$$