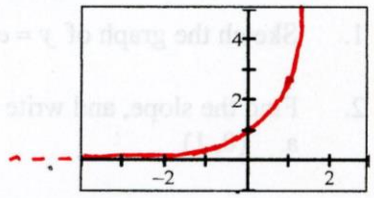
LESSON 7-2 DIFFERENTIATING AND INTEGRATING EXPONENTIAL FUNCTIONS



Use your calculator to graph $y = e^x$ and its derivative in the same coordinate plane. What do you notice?

They are the same.

e is the only base for which the basic exponential function and its derivative are the same. For other bases, a logarithmic "hook-on" is required.

Differentiating Exponential Functions:

$\frac{d}{dx} e^x = e^x$ $\qquad$ $\frac{d}{dx} e^u = e^u u'$ (Chain rule form), where u is a function of x. (u' is the "hook-on" factor)

$\frac{d}{dx} a^x = a^x \ln a$ $\qquad$ $\frac{d}{dx} a^u = a^u u' \ln a$ (u' and $\ln a$ are the "hook-on factors")

Examples: Differentiate

1. $y = e^{x^2-3x}$
$y' = e^{x^2-3x}(2x-3)$

2. $g(t) = e^{\frac{-3}{t}} = e^{-3t^{-1}}$
$g'(t) = e^{-3/t}(3t^{-2})$

3. $f(v) = 3^{\sqrt{v}} = 3^{v^{1/2}}$
$f'(v) = 3^{\sqrt{v}}\left(\frac{1}{2}v^{-\frac{1}{2}}\right)\ln 3$

4. Find the relative extrema of $f(x) = -xe^{-2x^2}$. List as points and do not use a calculator.

$f'(x) = -xe^{-2x^2}(-4x) + e^{-2x^2}(-1) = \underbrace{e^{-2x^2}}_{\text{(ALWAYS +)}}(4x^2-1)$

f' is never undefined

$f' = 0$ when $4x^2 - 1 = 0$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

f': $+$ $\quad -\frac{1}{2} \quad$ $-$ $\quad \frac{1}{2} \quad$ $+$

f

Rel. max: $\left(-\frac{1}{2}, \frac{1}{2}e^{-\frac{1}{2}}\right)$

Rel. min: $\left(\frac{1}{2}, -\frac{1}{2}e^{-\frac{1}{2}}\right)$

Integrating Exponential Functions:

$\int e^x\,dx = e^x + C$ $\qquad$ $\int e^u u'\,dx = e^u + C$ (Reverse Chain Rule form)

$\int a^x\,dx = \frac{a^x}{\ln a} + C$ $\qquad$ $\int a^u u'\,dx = \frac{a^u}{\ln a} + C$

Examples: Find

5. $\frac{1}{2}\int_0^{\frac{1}{2}} 4^{x^2-1} x\,dx \cdot 2$ $\quad$ $u = x^2-1$, $u' = 2x$

$\frac{1}{2}\cdot\frac{4^{x^2-1}}{\ln 4}\Big|_0^{\frac{1}{2}}$

$= \frac{1}{2}\cdot\frac{4^{-3/4}}{\ln 4} - \frac{1}{2}\cdot\frac{4^{-1}}{\ln 4}$

6. $\int \frac{e^{\frac{2}{x}}}{3x^2}\,dx = \frac{1}{3}\int e^{2x^{-1}}\cdot x^{-2}\,dx$ $\quad$ $u = 2/x = 2x^{-1}$, $u' = -2x^{-2}$

$= \frac{1}{3}\left(-\frac{1}{2}\right)\int e^{2x^{-1}}(-2x^{-2})\,dx$

$= -\frac{1}{6}e^{2/x} + C$

7. $\int \frac{e^x - e^{-x}}{\left(e^x + e^{-x}\right)^4}\,dx$

$\int (e^x + e^{-x})^{-4}(e^x - e^{-x})\,dx$

$= \frac{(e^x + e^{-x})^{-3}}{-3} + C$