## LESSON 7-1 EXPONENTIAL FUNCTIONS

An **exponential function** is a function represented by a constant base with a variable exponent. For example, $f(x)=2^x$, $y=e^x$, and $g(x)=3^{x^2-5}$ are exponential functions. These basic properties of exponents are used when working with exponential functions.

For $a$ and $b$ positive real numbers and $x$ and $y$ any real numbers:

1. $a^0=1$
2. $a^x a^y = a^{x+y}$
3. $\dfrac{a^x}{a^y} = a^{x-y}$
4. $\left(a^x\right)^y = a^{xy}$
5. $(ab)^x = a^x b^x$
6. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$
7. $a^{-x} = \dfrac{1}{a^x}$

Note: $(a+b)^x \neq a^x + b^x$

When simplifying, do not leave answers with negative exponents.

Examples: Simplify without using a calculator.

1. $27^{\frac{4}{3}}$

Note: $27^{\frac{4}{3}} = \sqrt[3]{27^4} = \left(\sqrt[3]{27}\right)^4 = 3^4 = 81$

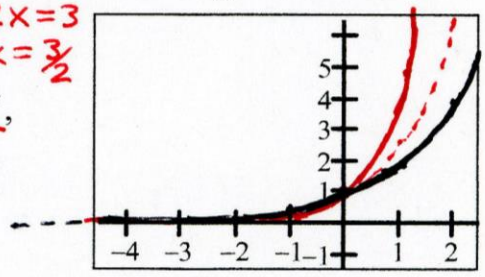
2. $\left(e+\dfrac{1}{e}\right)^0 = 1$

3. $\left(\dfrac{e^5 \cdot e^{-3}}{e^4}\right)^2 = \left(\dfrac{1}{e^2}\right)^2 = \dfrac{1}{e^4}$

4. $5^3 \cdot 25^{-2} = 5^3 (5^2)^{-2}$
$= 5^3 \cdot 5^{-4} = 5^{-1} = \dfrac{1}{5}$

5. Solve $9^x = 27$ without using a calculator.
$(3^2)^x = 3^3$
$3^{2x} = 3^3$
$2x = 3$
$x = \frac{3}{2}$

6. Use a calculator to carefully graph $y=2^x$, $y=5^x$, and $y=e^x$ in the same coordinate plane. Do you see any similarities in the graphs?

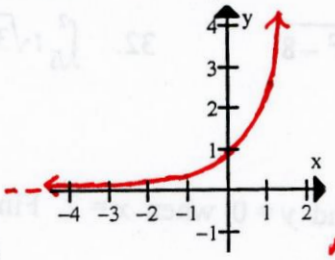



**Graphs of Exponential Functions**: If $f(x)=a^x$ and $a>1$, then

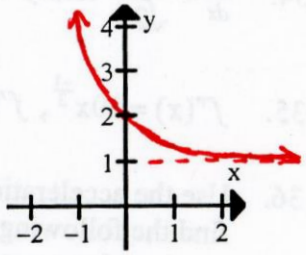
1. The domain of $f(x)$ is $(-\infty, \infty)$. The range of $f(x)$ is $(0, \infty)$.
2. The graph of $f(x)$ is continuous, increasing, concave upward, and one-to-one (has an inverse function).
3. The $x$-axis is a horizontal asymptote to the left: $\lim\limits_{x\to-\infty} f(x)=0$
(Also, $\lim\limits_{x\to\infty} f(x)=\infty$)
4. The $y$-intercept is (0, 1). Another key point is (1, $a$).

The letter $e$ used as a base in Examples 2, 3, and 6, is not an unknown. It is a number called the natural base for exponential functions. It is the most common base in Calculus, because functions with base $e$ are easier to differentiate and integrate than functions with other bases. By definition, $e = \lim_{x\to 0}(1+x)^{\frac{1}{x}}$. To three decimal places, $e \approx 2.718$.

Example 7: Without using a calculator, sketch a graph of $y = e^x$.




Example 8: Using adjustments to the graph from Example 7, graph $f(x) = e^{-x} + 1$ without using a calculator. Write an equation for the graph's asymptote.




Asy: $y = 1$

**ASSIGNMENT** 7-1

Simplify without a calculator.

1. $8^{\frac{2}{3}}$
2. $25^{\frac{-3}{2}}$
3. $3^0 - 5^0$
4. $(3-5)^0$
5. $\dfrac{4}{4^3}$
6. $(3^{-2})^{-1}$
7. $(3^{-4})(9^3)$
8. $\dfrac{8^2}{4^3}$
9. $\left(\dfrac{2}{e}\right)^{-3}$
10. $\left(\dfrac{-e^2}{e^{-2}}\right)^2$
11. $\left(\dfrac{e^2 \cdot e^{-1}}{e^{-4}}\right)^3$
12. $(e+3)^2$

Solve for $x$ without a calculator.

13. $2^x = 16$
14. $3^{2x-3} = 27$
15. $\left(\dfrac{1}{2}\right)^{2x} = 8$
16. $x^{\frac{4}{3}} = 16$
17. $\left(\dfrac{e^2}{e^x}\right)^3 = e^9$
18. $(5-e)^x = 1$

For Problems 19-24, sketch a graph without using a calculator. List all intercepts, and write an equation for each asymptote. Use a separate coordinate plane for each graph.

19. $y = 2^x$
20. $y = 2^{-x}$
21. $y = -2^x$
22. $y = 2^{x+1}$
23. $y = 2^x - 1$
24. $y = 2^{|x|}$

25. Find the average value of $f(x) = \dfrac{1}{\sqrt{2-x}}$ on the interval $[-2,1]$.

26. Sketch the region bounded by $x = y^2 - y$ and $x = 0$, and find the area of the region. (Show an integral set-up first.)

27. For $f(x) = x^3$, find the value of $c$ in $[-2,0]$ such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.