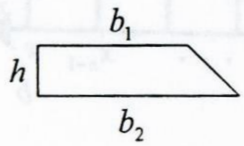
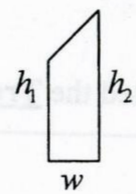
LESSON 6-3 TRAPEZOIDAL RULE

For most functions, using trapezoids to approximate "areas" is more accurate than using rectangles.

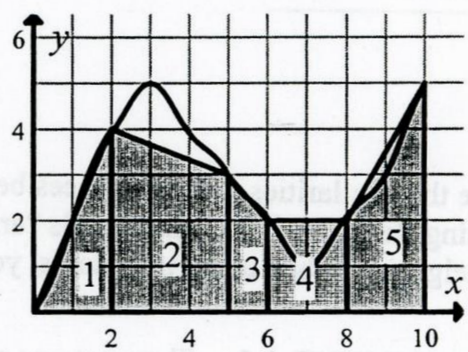
The area formula for a single trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. Figure (below left).





Since trapezoids used in the approximations of "areas" are usually positioned vertically, we will write the formula as $A = \frac{1}{2}w(h_1 + h_2)$. Figure (above right).

To approximate the value of a definite integral using trapezoids, use the same strategy as you used for Riemann Sums – but add the "areas" of trapezoids instead of rectangles.



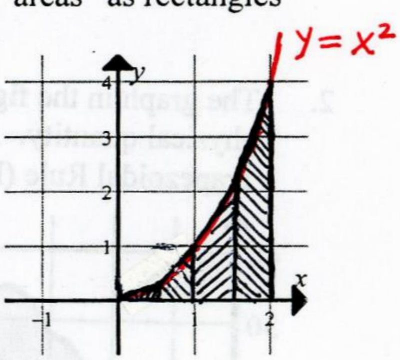
Example 1: Approximate $\int_0^{10} f(x)\,dx$ by adding the areas of the 5 "trapezoids" shown in the graph at the right.

$\int_0^{10} f(x)dx \approx$

$\frac{1}{2}\cdot 2(0+4)+\frac{1}{2}\cdot 3(4+3)+\frac{1}{2}\cdot 1(3+2)+\frac{1}{2}\cdot 2(2+2)$
$+\frac{1}{2}\cdot 2(2+5)$

Note: The area formula for a trapezoid also works for a triangle (either $h_1 = 0$ or $h_2 = 0$) or a rectangle $(h_1 = h_2)$.

If you forget the formula for a trapezoid, you can always draw your "areas" as rectangles and triangles (and not even use trapezoids).



Example 2: Use four trapezoids of equal width to find an approximation for the area under the curve $y = x^2$, bounded below by the x-axis, on the left by $x = 0$, and on the right by $x = 2$. Sketch the curve and draw the trapezoids first.

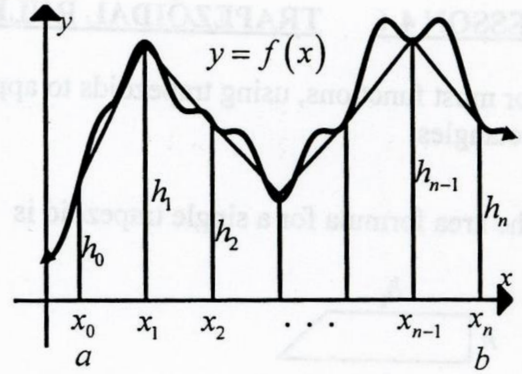
$A \approx \frac{1}{2}\cdot\frac{1}{2}(0+2\cdot\frac{1}{4}+2\cdot 1+2\cdot\frac{9}{4}+4)$

When using trapezoids for approximations, if the subdivisions produce equal widths (see the figure at the right), then

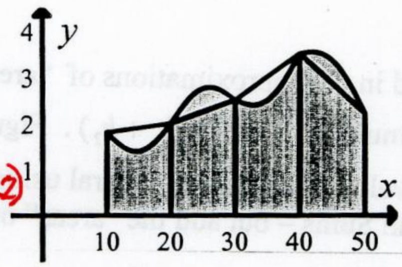
$$\int_a^b f(x)\,dx \approx \frac{1}{2}w(h_0 + 2h_1 + 2h_2 + \ldots + 2h_{n-1} + h_n)$$

where $w = \dfrac{b-a}{n}$

This formula is called the **Trapezoidal Rule**.



Example 3: Use the Trapezoidal Rule with four equal subdivisions to approximate the area shown.



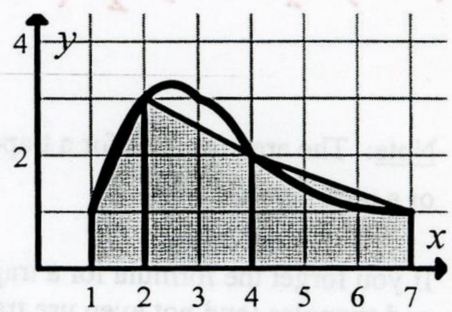
x	y
10	1.8
20	2
30	2.5
40	3.5
50	2.2

$A \approx \frac{1}{2}\cdot 10(1.8 + 2\cdot 2 + 2\cdot 2.5 + 2\cdot 3.5 + 2.2)$

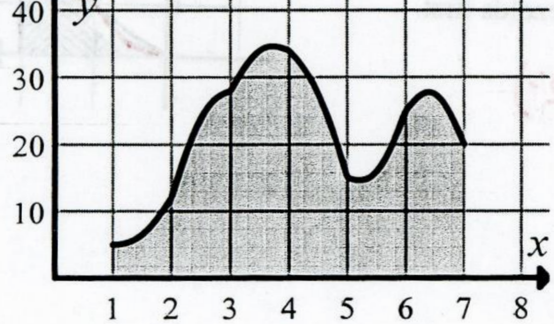
Note the similarities and differences between the Trapezoidal Rule and the formula for finding Riemann Sums. Each adds "areas" by multiplying a "common width" by a "sum of heights." When using trapezoids, you have Half the Width, but Twice the Heights.

ASSIGNMENT 6-3 Show set ups on Problems 1-9.

1. Use the trapezoids shown to approximate $\int_1^7 f(x)\,dx$.



2. The graph in the figure below was recorded by an instrument used to measure a physical quantity. Approximate the area of the shaded region by using the Trapezoidal Rule (let $n = 6$).



x	y
1	5
2	12
3	28
4	34
5	15
6	25
7	20