LESSON 6-2 RIEMANN SUMS

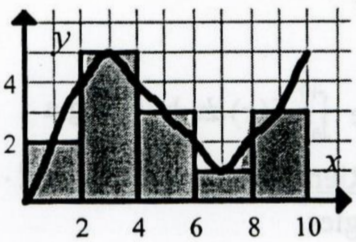
In Lesson 4-3, you learned how to find "signed areas" using definite integrals. Not all such "areas" can be found this way. Some functions cannot be integrated (they are said to be non-integrable), and sometimes you are given data or a graph – but not an actual function. It is still possible to approximate "areas." In this lesson, you will approximate "areas" by adding "areas" of rectangles. This sum is called a **Riemann Sum**.

Example 1:

Approximate $\int_0^{10} f(x)\,dx$ by adding the areas of the five rectangles shown.

This is a Midpoint Riemann Sum using five subdivisions of equal width. The heights of the rectangles are determined from the midpoints of intervals. Be sure to use $f(x)$ values (not x-values) for your heights.

$y = f(x)$



x	$f(x)$
0	0
1	2
2	4
3	5
4	4
5	3
6	2
7	1
8	2
9	3
10	5

$$\int_0^{10} f(x)dx \approx 2\cdot2 + 2\cdot5 + 2\cdot3 + 2\cdot1 + 2\cdot3$$
$$= 4 + 10 + 6 + 2 + 6 = 28$$
$$\text{or } 2(2+5+3+1+3) = 28$$

In many cases, the rectangles used to approximate "areas" are of equal width. In such cases, you can use a **formula to find the Riemann Sum** used to approximate the "area."

$$\int_a^b f(x)\,dx \approx w\underbrace{(h_1 + h_2 + h_3 + \ldots + h_n)}_{\text{Sum of "heights" of rectangles.}}$$

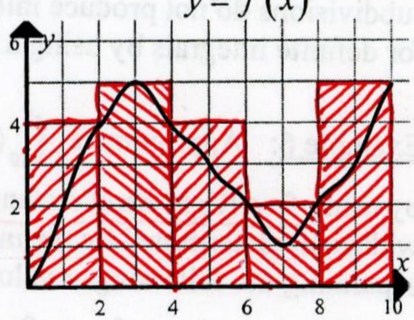
Note: The number of "heights" will equal the number of rectangles.

$w = \dfrac{b-a}{n}$ = the width of each rectangle. n = number of rectangles.

Example 2:

Approximate $\int_0^{10} f(x)\,dx$ by using 5 rectangles of equal width ($n = 5$) and an Upper (Circumscribed) Riemann Sum. The heights of the rectangles are determined by the upper (largest) $f(x)$ value in each interval. Draw rectangles on the figure.

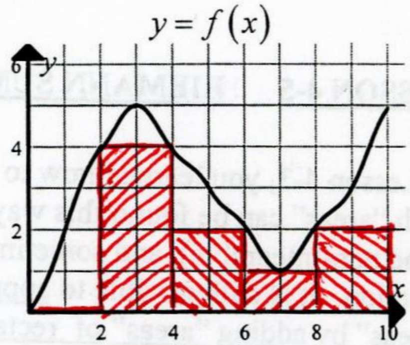
$y = f(x)$



$$\int_0^{10} f(x)dx \approx 2(4+5+4+2+5) = 40$$

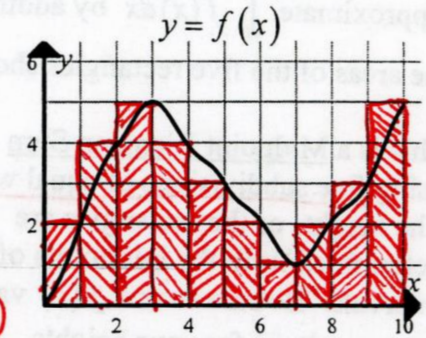
Examples:

3. Approximate $\int_0^{10} f(x)\,dx$ by using a Lower (Inscribed) Riemann Sum with $n=5$. Draw rectangles.



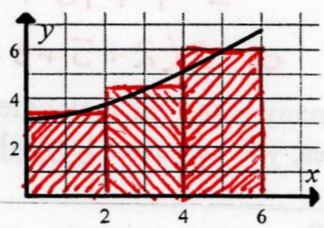
$\int_0^{10} f(x)dx \approx 2(0+4+2+1+2) = 18$

4. Approximate $\int_0^{10} f(x)\,dx$ by using a Right hand Riemann Sum with $n=10$. Draw rectangles.



$\int_0^{10} f(x)dx \approx 1(2+4+5+4+3+2+1+2+3+5)$
$= 31$

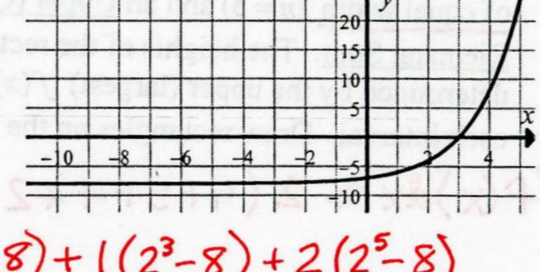
5. Approximate $\int_0^{6} \sqrt{x^2+10}\,dx$ using a Midpoint Riemann Sum with 3 equal subdivisions.



$\int_0^6 \sqrt{x^2+10}\,dx \approx 2(\sqrt{11}+\sqrt{19}+\sqrt{35})$

It is possible that $f(x)$ is not always greater than or equal to zero and/or that your subdivisions do not produce intervals of equal width. You can still approximate values for definite integrals by using a Riemann Sum.

Example 6: Approximate $\int_{-10}^{5} (2^x - 8)\,dx$ by using five Right hand rectangles whose widths are determined by the intervals separating the following x values:
$x=-10$, $x=-4$, $x=0$, $x=2$, $x=3$, and $x=5$.



$\int_{-10}^{5}(2^x-8)dx \approx 6(2^{-4}-8)+4(2^0-8)+2(2^2-8)+1(2^3-8)+2(2^5-8)$