LESSON 6-1 THE SECOND FUNDAMENTAL THEOREM OF CALCULUS, INTERPRETATION OF "RATE" GRAPHS

Remember that $\frac{d}{dx}\int(2x-3)\,dx = 2x-3$.

The following examples serve as an informal guide toward discovering the Second Fundamental Theorem of Calculus.

1. $\int_0^x (2t-3)\,dt = (t^2-3t)\Big|_0^x = x^2-3x$

2. $\frac{d}{dx}\int_0^x (2t-3)\,dt = 2x-3$

Do you notice anything yet?

3. $\int_{10}^x f'(t)\,dt = f(t)\Big|_{10}^x = f(x)-f(10)$

4. $\frac{d}{dx}\int_{10}^x f'(t)\,dt = f'(x)$

How about now?

Second Fundamental Theorem of Calculus:

For any constant a, $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ (if f is continuous from a to x).

Now find:

5. $\int_0^{x^2} f'(t)\,dt = f(t)\Big|_0^{x^2}$
$= f(x^2) - f(0)$

6. $\frac{d}{dx}\int_0^{x^2} f'(t)\,dt = f'(x^2)\cdot 2x$

7. $\int_{x^3}^{2x} f'(t)\,dt = f(t)\Big|_{x^3}^{2x}$
$= f(2x) - f(x^3)$

8. $\frac{d}{dx}\int_{x^3}^{2x} f'(t)\,dt =$
$f'(2x)\cdot 2 - f'(x^3)\cdot 3x^2$

What changes did you notice?

Second Fundamental Theorem (Chain Rule Version):

If u and v are functions of x, then $\frac{d}{dx}\int_u^v f(t)\,dt = f(v)\,v' - f(u)\,u'$

(if f is continuous from u to v). Note the "hook-ons" v' and u'.

Examples: Find each of the following without integrating.

9. $\frac{d}{dx}\int_x^0 (2t-3)\,dt = 0-(2x-3)$
$= -2x+3$

10. $\frac{d}{dx}\int_2^5 (2t-3)\,dt = 0-0=0$

11. $\frac{d}{dx}\int_{-1}^{x^3} (t^2+2t)\,dt =$
$(x^6+2x^3)\cdot 3x^2 - 0$

12. $\frac{d}{dx}\int_{2x}^{3x} (t^2+2t)\,dt = (9x^2+6x)\cdot 3-(4x^2+4x)\cdot 2$

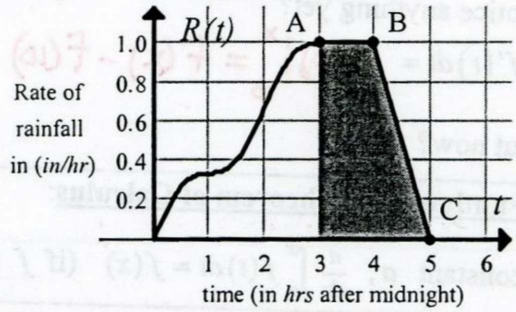
13. $\frac{d}{dx}\int_{f(x)}^{g(x)} (2t-3)\,dt =$
$(2g(x)-3)\cdot g'(x)-(2\cdot f(x)-3)\cdot f'(x)$

14. If $f(x) = \int_0^{3x^2} (1-t^2)^{10}\,dt$, then $f'(x) =$
$(1-9x^4)^{10}\cdot 6x - 0$

Interpretation of "Rate" Graphs:

Graphs of "rates" are common in Calculus. While the horizontal axis in such graphs may stand for many things, it often stands for time. The vertical axis stands for some indicated type of rate (often with respect to time). In **"Rate" graphs**,
a Point on the graph represents a value of the indicated rate **at** a certain time: ___ per ___,
a Slope (derivative) represents a rate of change of that indicated rate: ___ per ___ per ___,
and the "Signed Area" between the graph and the horizontal (time) axis represents a definite integral which has "undone" the indicated rate to produce an "accumulated value." In this context, an integral may be thought of as an accumulator.

Examples:
The graph at right models the rate of rainfall in inches per hour from midnight until 6:00 A.M. during a tropical rainstorm.



15. Write a complete sentence to explain what Point A on the graph represents. Include numbers and units in your answer to this Example and Examples 17, 19, 21, and 23.

At 3 A.M., it is raining at a rate of 1 inch/hour.

16. What is the slope of the graph between Points A and B? 0

17. Write a complete sentence to explain the meaning of your answer to Example 16.

Between 3 A.M. and 4 A.M, there is no change in the rate of rainfall. (The rate of rainfall stays at a constant 1 inch/hour.)

18. What is the slope of the graph between Points B and C? -1

19. Write a complete sentence to explain the meaning of your answer to Example 18.

From 4 A.M. to 5 A.M., the rate of rainfall decreases at a rate of 1 inch/hour/hour.

20. Find $\int_3^5 R(t)\,dt$. $1\cdot 1 + \frac{1}{2}\, 1\cdot 1 = 1\frac{1}{2}$

21. Write a complete sentence to explain the meaning of your answer to Example 20.

During the tropical rainstorm, it rained a total of $1\frac{1}{2}$ inches from 3 A.M. to 5 A.M.

22. Approximate the value of $\int_0^6 R(t)\,dt$ using geometrical regions. Show computations.

$1\frac{1}{2} + \frac{1}{2}\cdot 3 \cdot 1 = 3$

23. Write a complete sentence to explain the meaning of your answer to Example 22.

During the tropical rainstorm, approximately 3 inches of rain fell between midnight and 6 A.M.