LESSON 5-4 ANTIDIFFERENTIATION, INDEFINITE INTEGRALS

Warm-up Examples: Differentiate each of the following.

1. $f(x) = x^3$
   $f'(x)=3x^2$

2. $f(x) = x^3 - 10$
   $f'(x)=3x^2$

3. $f(x) = x^3 + C$
   where C is any constant (number)
   $f'(x) = 3x^2$

So what should you get when you antidifferentiate $f'(x) = 3x^2$? $f(x) =$ $x^3 + C$

This problem can be written as $\int 3x^2 dx =$ $x^3 + C$

The symbol $\int$ is called an integral symbol and tells you to integrate (antidifferentiate) the expression which follows it. That expression is called an integrand. dx indicates that you are integrating with respect to the variable x but does not affect the integration process. C is called the constant of integration and must be written as part of your answer when you are antidifferentiating.

Integration Rules:

Power Rule:	$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C, \quad n \neq -1$
Constant Rule:	If k is any constant, $\int k\,dx = kx + C$
Scalar Multiple Rule:	If k is any constant, $\int k\,f(x)\,dx = k\int f(x)\,dx$ (Constants may be "factored out" of the integral expression. **NEVER** "factor out" a variable.)
Sum Rule:	$\int \big(f(x) + g(x)\big)\,dx = \int f(x)\,dx + \int g(x)\,dx$

Examples: Evaluate (Integrate).

4. $\int x^3\,dx$
   $\dfrac{x^4}{4} + C$

5. $\int 2\,dx$
   $2x + C$

6. $\int (t^4 + 2)\,dt$
   $\dfrac{t^5}{5} + 2t + C$

7. $\int (2y^2 + 4y + 1)\,dy$

$\frac{2y^3}{3} + 2y^2 + y + C$

8. $\int \left(\frac{3}{x^2} - \frac{1}{\sqrt{x}} \right) dx$

$\int (3x^{-2} - x^{-\frac{1}{2}})\,dx$

$-3x^{-1} - 2x^{\frac{1}{2}} + C$

or $-\frac{3}{x} - 2\sqrt{x} + C$

9. $\int \frac{\sqrt{x}+1}{x^2}\,dx$

$\int \left(\frac{\sqrt{x}}{x^2} + \frac{1}{x^2} \right) dx$

$\int (x^{-\frac{3}{2}} + x^{-2})\,dx$

$-2x^{-\frac{1}{2}} - x^{-1} + C$

Note: Put $+C$ when you integrate, but never when you differentiate.

Sometimes an initial condition is given which makes it possible to solve for C.

Example 10: If $f'(x) = x^{-3}$ and $f(1) = \frac{3}{2}$, find $f(x)$.

$f(x) = \frac{x^{-2}}{-2} + C$

$f(x) = -\frac{1}{2x^2} + C$

since $f(1) = \frac{3}{2}$ $\quad \frac{3}{2} = -\frac{1}{2} + C$

$2 = C$

$\boxed{f(x) = -\frac{1}{2x^2} + 2}$

Example 11: Evaluate $\frac{d}{dx} \int \left(5\sqrt{x} - \frac{1}{x^2} \right) dx$

$5\sqrt{x} - \frac{1}{x^2}$

If we know the acceleration equation for an object, and if we are given initial conditions for the object's velocity and position, integration allows us to find the velocity and position equations for the object.

Remember: Pos. $\rightarrow$ Vel. $\rightarrow$ Acc. (Differentiate), so Acc. $\rightarrow$ Vel. $\rightarrow$ Pos. (Integrate).

Example 12: The acceleration of a particle at time t is given by $a(t) = 4t - 3$. $v(1) = 6$ and $s(2) = 5$.

a. Find the velocity equation. $v(t) = 2t^2 - 3t + C_1$

since $v(1) = 6$ $\quad 6 = 2 - 3 + C_1$

$7 = C_1$

$v(t) = 2t^2 - 3t + 7$

b. Find the position equation. $s(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 + 7t + C_2$

since $s(2) = 5$ $\quad 5 = \frac{2}{3} \cdot 8 - \frac{3}{2} \cdot 4 + 7 \cdot 2 + C_2$

$5 = \frac{16}{3} - 6 + 14 + C_2$

$-3 - \frac{16}{3} = C_2$

$-\frac{25}{3} = C_2$

$s(t) = \frac{2}{3}t^3 - \frac{3}{2}t^2 + 7t - \frac{25}{3}$

Example 13: Given that on earth, the acceleration of an object due to gravity is approximately $-32\ ft/sec^2$ (negative indicates downward), develop

a. the equation for the velocity of the object. v_o = initial velocity → when $t=0$, $v = v_0$

$v(t) = -32t + C_1$

$v_0 = -32 \cdot 0 + C_1$

$v_0 = C_1$

$v(t) = -32t + v_0$

b. the equation for the position of the object. s_o = initial position → when $t=0$, $s = s_0$

$s(t) = -16t^2 + v_0 t + C_2$

$s_0 = -16 \cdot 0 + v_0 \cdot 0 + C_2$

$s_0 = C_2$

$s(t) = -16t^2 + v_0 t + s_0$

> Note: The two equations $v(t) = -32t + v_0$ and $s(t) = -16t^2 + v_0 t + s_0$ may be used for any motion affected only by the earth's gravity.

ASSIGNMENT 5-4

For Problems 1-4, rewrite the integrand and then integrate.

1. $\int \frac{1}{x^3}\,dx$ 2. $\int \sqrt[4]{t}\,dt$ 3. $\int (x+1)(x-2)\,dx$ 4. $\int \frac{2y}{\sqrt{y}}\,dy$

Evaluate (integrate) each integral in Problems 5-13.

5. $\int (2x^3 - x^2 + 1)\,dx$ 6. $\int \frac{1}{3x^2}\,dx$ 7. $\int \frac{1}{(3x)^2}\,dx$

8. $\int \left(3 - \sqrt[5]{y^2}\right) dy$ 9. $\int \left(5x^{\frac{1}{4}} - x^{\frac{-2}{3}}\right) dx$ 10. $\int (3t-10)^2\,dt$

11. $\int \frac{8x^4 - 2x^2 + 1}{2x^2}\,dx$ 12. $\int \frac{2\sqrt{t} - 1}{\sqrt{t}}\,dt$ 13. $\int \sqrt{y}\left(y^2 + 2\sqrt{y}\right) dy$

14. If $f'(x) = 3x^2 - 4x + 2$ and $f(1) = -3$, find $f(x)$.