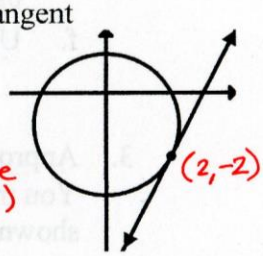
LESSON 5-3 APPROXIMATING WITH THE TANGENT LINE

In many instances, finding a value of a function is difficult or impossible. With the use of Calculus techniques, we can approximate the function value by finding a y-value on a tangent line to the function. Since this method involves using a linear function (the tangent line function) at a nearby point, it is sometimes called a local linearization approximation.

Examples:

1. If $(2,-2)$ is a point on the graph of $x^2+y^2+2y=4$, use the equation of a tangent line passing through the point $(2,-2)$ to approximate a y-coordinate



(a) when the x-coordinate is 2.1.

$2x+2yy'+2y'=0$

$2yy'+2y'=-2x$

$y'(2y+2)=-2x$

$y'=\frac{-2x}{2y+2}$

$y'(2,-2)=\frac{-4}{-2}=2=m_{tan.}$

T.L: $y+2=2(x-2)$

$y=2(x-2)-2$ (tangent line y-value)

when $x=2.1$,

$y\approx 2(2.1-2)-2=2(.1)-2=.2-2=-1.8$

(-1.8 is the y-value on the tangent line, which approximates the actual y-value on the curve.)

(b) when the x-coordinate is 1.9.

$y\approx 2(1.9-2)-2=2(-.1)-2=-.2-2=-2.2$

2. If $f(2)=3$ and $f'(2)=-2$, use local linearization to approximate $f(2.01)$,

Point: $(2,3)$

$m_{tan}=-2$

T.L: $y-3=-2(x-2)$

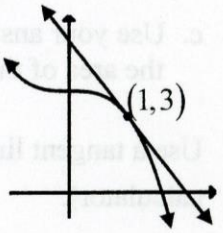
$y=-2(x-2)+3$

$f(2.01)\approx -2(2.01-2)+3=-2(.01)+3=-.02+3$

$=2.98$

ASSIGNMENT 5-3

$f(x)=-x^3+4$



1. a. Write an equation of the tangent line shown.
   b. Use this tangent line equation to approximate $f(1.1)$.
   c. What is the actual value of $f(1.1)$?