LESSON 4-4 CURVE SKETCHING WITH EXTREMA AND POINTS OF INFLECTION

You used a Precalculus Curve Sketching Recipe in Unit 1. The following incorporates the additional Calculus techniques you have recently learned.

Curve Sketching Recipe:

1. Give the domain.
2. Reduce $f(x)$.
3. Find vertical asymptotes and holes.
4. Give x- and y-intercepts.
5. Find the end behavior (horizontal asymptotes or other).
6. (optional) Check for symmetry.
7. Find increasing/decreasing intervals and relative extrema (show an f' number line).
8. Find concavity and points of inflection (show an f'' number line).
9. Graph.

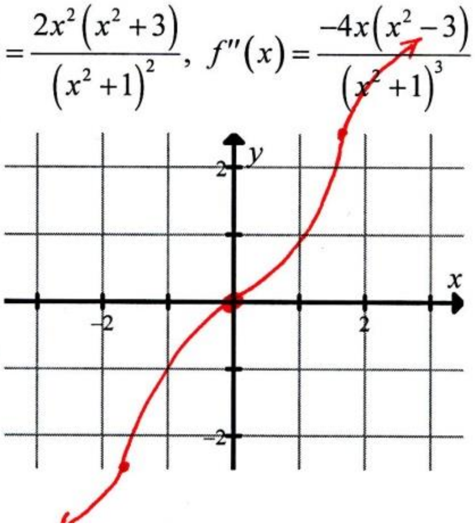
Examples:

1. (a rational function) $f(x)=\frac{2x^3}{x^2+1}$, $f'(x)=\frac{2x^2(x^2+3)}{(x^2+1)^2}$, $f''(x)=\frac{-4x(x^2-3)}{(x^2+1)^3}$

Do.: all reals
V.A.: none
Holes: none
x-int.: (0,0) odd
y-int.: (0,0)
E.B.: like $y=2x$ ↙↗
Symmetry: origin
Rel. Max.: } none
Rel. Min.: } none
P.I.: $(0,0)$ $\left(\sqrt{3}, \frac{3\sqrt{3}}{2}\right)$ $\left(-\sqrt{3}, \frac{-3\sqrt{3}}{2}\right)$

<u>CN</u> $x=0$
f' ← + | + →
0

PPI $x=0, \pm\sqrt{3}$
f'' ← + | − | + | − →
$-\sqrt{3}$ 0 $\sqrt{3}$



2. (a rational function) $f(x)=\frac{3x-2}{x^2-2x+1}$, $f'(x)=\frac{-3x+1}{(x-1)^3}$, $f''(x)=\frac{6x}{(x-1)^4}$

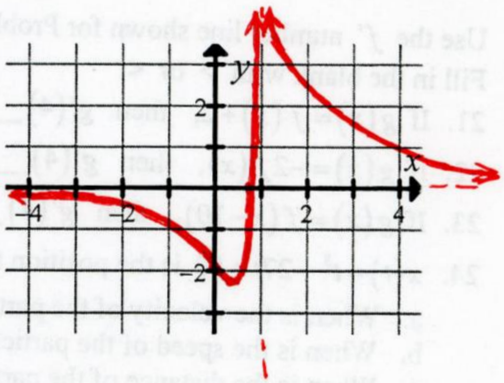
$(x-1)^2$

PRE {
Do.: $x \neq 1$
V.A.: $x=1$ even
Holes: none
x-int.: $(2/3, 0)$
y-int.: $(0,-2)$
E.B.: $y=0$ (H.A.)
Symmetry: none
}
f' Rel. Max.: none
f' Rel. Min.: $\left(\frac{1}{3}, \frac{-1}{\frac{4}{9}}\right)$ or $\left(\frac{1}{3}, -\frac{9}{4}\right)$
f'' P.I.: $(0,-2)$

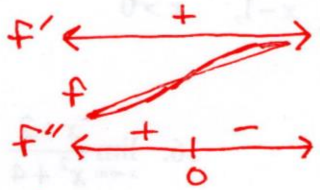
f' ← − | + ⋮ − →
$\frac{1}{3}$

f'' ← − | + ⋮ + →
0

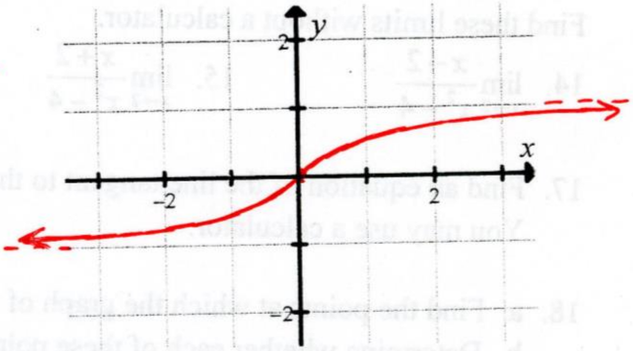
f



3. (a radical function) $f(x)=\dfrac{x}{\sqrt{x^2+2}}$, $f'(x)=\dfrac{2}{\sqrt{\left(x^2+2\right)^3}}$, $f''(x)=\dfrac{-6x}{\sqrt{\left(x^2+2\right)^5}}$



Do.: Reals
V.A.: none
Holes: none
x-int.: (0,0) odd
y-int.: (0,0)
E.B.: $y=\frac{x}{|x|}$ R: $y=1$ L: $y=-1$
Symmetry: origin
Rel. Max.: none
Rel. Min.: none
P.I.: (0,0)



ASSIGNMENT 4-4

Without using a calculator, find local extrema, points of inflection, and sketch a graph. Show organized steps and justification. It is not necessary to find x-intercepts and there are no domain restrictions or asymptotes. However, an end behavior analysis will be helpful.

1. $y=x^3-3x^2+5$ 2. $y=1-x-x^3$ 3. $y=x^4-4x^3+16$

Find intercepts and relative extrema and graph these functions without using a calculator.

4. $f(x)=x^2-2x-8$ 5. $g(x)=\left|x^2-2x-8\right|$

Find relative extrema, points of inflection, and end behavior and graph without a calculator.

6. $y=\dfrac{2x^2}{x^2+3}$, $y''=\dfrac{-36\left(x^2-1\right)}{\left(x^2+3\right)^3}$

Find the domain, relative extrema, asymptotes, and end behavior and graph without a calculator. There are no points of inflection.

7. $f(x)=\dfrac{x^2+1}{2x}$

Find the domain, reduced function, hole, intercepts, relative extrema, and points of inflection. Then graph without using a calculator.

8. $f(x)=\dfrac{x^2\sqrt{4-x}}{x}$, $f'=\dfrac{8-3x}{2\sqrt{4-x}}$, $f''=\dfrac{3x-16}{4(4-x)^{\frac{3}{2}}}$, $f\left(\frac{8}{3}\right)=3.079$

9. Without a calculator, find the <u>domain</u>, x-intercepts, and relative extrema. Then graph $f(x)$. There are no points of inflection. $f(x)=\sqrt{9-x^2}$

10. True or False? If $f'(x)>0$ for all real x-values, then $\lim\limits_{x\to\infty} f(x)=\infty$.

Show a graph to illustrate your answer.