

**LESSON 4-2 INCREASING/DECREASING FUNCTIONS,
FIRST DERIVATIVE TEST FOR RELATIVE EXTREMA**

PROCEDURE: (Increasing/ Decreasing and First Derivative Test)

1. Find **domain** restrictions.
2. Find all **critical numbers** (where $f'(x) = 0$ or $f'(x)$ is undefined – but domain restrictions cannot be critical numbers).
3. Locate critical numbers and domain restrictions on an f' **number line**. Label critical numbers CN.
4. Test the sign of $f'(x)$ in each interval and **label the signs** on the number line.
5. List **increasing/decreasing intervals** and/or identify **relative min/max** x -values. If requested, find y -values or points.

Examples: Find the intervals on which these functions are increasing and decreasing and find all local extrema points.

1. $f(x) = (x^2 - 9)^{\frac{2}{3}}$

D: Reals

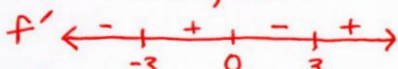
$$f'(x) = \frac{2}{3}(x^2 - 9)^{-\frac{1}{3}}(2x)$$

$$= \frac{4x}{3\sqrt[3]{x^2 - 9}}$$

$f'(x) = 0$ when $x = 0$

$f'(x)$ is undefined when $x = \pm 3$

C.N: $x = 0, \pm 3$



Inc: $(-3, 0), (3, \infty)$

Dec: $(-\infty, -3), (0, 3)$

Rel. min: $(\pm 3, 0)$

Rel. max: $(0, (-9)^{\frac{2}{3}})$

2. $f(x) = \frac{x^4 + 3}{3x}$

D: $x \neq 0$

$$f'(x) = \frac{3x(4x^3) - (x^4 + 3)(3)}{(3x)^2}$$

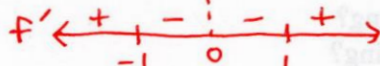
$$= \frac{12x^4 - 3x^4 - 9}{9x^2} = \frac{9x^4 - 9}{9x^2}$$

$$= \frac{9(x^4 - 1)}{9x^2} = \frac{(x^2 + 1)(x + 1)(x - 1)}{x^2}$$

$f'(x) = 0$ when $x = \pm 1$

$f'(x)$ is undefined when $x = 0$
(NOT a C.N.)

C.N: $x = \pm 1$



Inc: $(-\infty, -1), (1, \infty)$

Dec: $(-1, 0), (0, 1)$

Rel. min: $(1, \frac{1}{3})$

Rel. max: $(-1, -\frac{1}{3})$

3. $f(x) = x^3 - 3x^2 + 3x$

D: Reals

$$f'(x) = 3x^2 - 6x + 3$$

$$= 3(x^2 - 2x + 1)$$

$$= 3(x - 1)^2$$

$f'(x)$ is never undefined

C.N: $x = 1$



Inc: $(-\infty, \infty)$

Dec: never

no rel. extrema

The function in Example 3 is a strictly monotonic function. Strictly increasing or strictly decreasing functions are called monotonic.

Example 3 illustrates two **important points**.

1. Not every critical number produces a relative maximum or minimum.
2. Even though the slope at $x = 1$ is zero, the function is always increasing.