LESSON 4-2 INCREASING/DECREASING FUNCTIONS, FIRST DERIVATIVE TEST FOR RELATIVE EXTREMA

PROCEDURE: **(Increasing/ Decreasing** and **First Derivative Test)**

1. Find **domain** restrictions.
2. Find all **critical numbers** (where $f'(x) = 0$ or $f'(x)$ is undefined – but domain restrictions cannot be critical numbers).
3. Locate critical numbers and domain restrictions on an f' **number line.** Label critical numbers CN.
4. Test the sign of $f'(x)$ in each interval and **label the signs** on the number line.
5. List **increasing/decreasing intervals** and/or identify **relative min/max** x-values. If requested, find y-values or points.

Examples: Find the intervals on which these functions are increasing and decreasing and find all local extrema points.

1. $f(x) = (x^2-9)^{\frac{2}{3}}$

D: Reals

$f'(x) = \frac{2}{3}(x^2-9)^{-\frac{1}{3}}(2x)$

$= \dfrac{4x}{3\sqrt[3]{x^2-9}}$

$f'(x) = 0$ when $x = 0$

$f'(x)$ is undefined when $x = \pm 3$

C.N: $x = 0, \pm 3$

f': $-$ | $+$ | $-$ | $+$ at -3, 0, 3

f

Inc: $(-3, 0)$, $(3, \infty)$

Dec: $(-\infty, -3)$, $(0, 3)$

Rel. min: $(\pm 3, 0)$

Rel. max: $(0, (-9)^{2/3})$

2. $f(x) = \dfrac{x^4+3}{3x}$

D: $x \neq 0$

$f'(x) = \dfrac{3x(4x^3) - (x^4+3)(3)}{(3x)^2}$

$= \dfrac{12x^4 - 3x^4 - 9}{9x^2} = \dfrac{9x^4 - 9}{9x^2}$

$= \dfrac{\cancel{9}(x^4-1)}{\cancel{9}x^2} = \dfrac{(x^2+1)(x+1)(x-1)}{x^2}$

$f'(x) = 0$ when $x = \pm 1$

$f'(x)$ is undefined when $x = 0$ (NOT a C.N.)

C.N: $x = \pm 1$

f': $+$ | $-$ ¦ $-$ | $+$ at -1, 0, 1

f

Inc: $(-\infty, -1)$, $(1, \infty)$

Dec: $(-1, 0)$, $(0, 1)$

Rel. min: $(1, \frac{4}{3})$

Rel. max: $(-1, -\frac{4}{3})$

3. $f(x) = x^3 - 3x^2 + 3x$

D: Reals

$f'(x) = 3x^2 - 6x + 3$

$= 3(x^2 - 2x + 1)$

$= 3(x-1)^2$

$f'(x)$ is never undefined

C.N: $x = 1$

f': $+$ | $+$ at 1

f

Inc: $(-\infty, \infty)$

Dec: never

no rel. extrema

The function in Example 3 is a strictly monotonic function. Strictly increasing or strictly decreasing functions are called monotonic.

Example 3 illustrates two **important points.**

1. Not every critical number produces a relative maximum or minimum.
2. Even though the slope at $x = 1$ is zero, the function is always increasing.