LESSON 3-5 RELATED RATES

In related rate story problems, the idea is to find a rate of change (with respect to time) of one quantity by using the rate of change (with respect to time) of a related quantity.

Procedure for Related Rate Problems

1. Draw a figure (if necessary) and choose variables for all unknowns.
2. Write what is given and what is to be found using your variables and $\frac{d}{dt}$ symbols.
3. Write an equation relating the variables.
   (a) If a quantity is **changing** it must be represented with a variable letter.
   (b) If a quantity is **constant** it must be represented with a number value.
   (c) Look for secondary relationships between quantities to reduce the number of variables.
4. Implicitly differentiate both sides with respect to t.
5. Substitute number values and solve.

Geometry Formulas

Right Triangle	Circle	Sphere	Cube	Cone	Rectangle
$a^2+b^2=c^2$	$A=\pi r^2$	$V=\frac{4}{3}\pi r^3$	$V=e^3$	$V=\frac{1}{3}\pi r^2 h$	$A=lw$
$A=\frac{1}{2}bh$	$C=2\pi r$	$A=4\pi r^2$	$A=6e^2$		$P=2l+2w$

Examples:

1. In these problems, the first three steps of the procedure are done. We need to complete only the last two steps.

   a. Given: $A=s^2,\ \frac{ds}{dt}=3,\ s=4$ Find: $\frac{dA}{dt}$

   $\frac{dA}{dt}=2s\frac{ds}{dt}$

   $\frac{dA}{dt}=2\bullet4\bullet3$

   $\frac{dA}{dt}=24$

   b. Given: $\frac{dx}{dt}=-6,\ y=4,\ \underbrace{xy}_{\text{Product}}=12$ Find: $\frac{dy}{dt}$

   $x\frac{dy}{dt}+y\frac{dx}{dt}=0$ $x\cdot4=12$ $x=3$

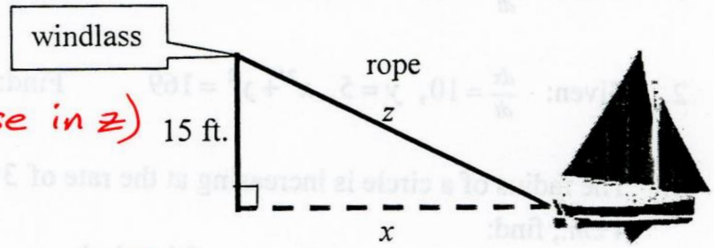
   For $y=4$, $x=3$: $3\frac{dy}{dt}+4(-6)=0$

   $3\frac{dy}{dt}-24=0$

   $3\frac{dy}{dt}=24$

   $\frac{dy}{dt}=8$

2. A windlass is used to tow a boat to a dock. The rope is attached to the boat at a point 15 feet below the level of the windlass. If the windlass pulls in rope at the rate of 30 feet per minute, at what rate is the boat approaching the dock when there is 25 feet of rope out?



Given: $\frac{dz}{dt} = -30$ ft./min.
(negative indicates decrease in z)
Find: $\frac{dx}{dt}$ when $z = 25$

$x^2 + 15^2 = z^2$

$2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$

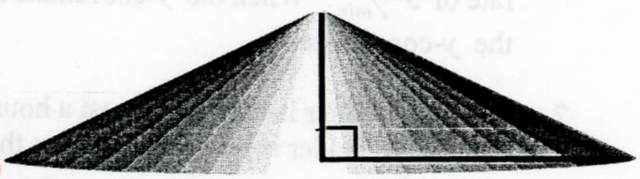
$x\frac{dx}{dt} = z\frac{dz}{dt}$

when $z = 25$,
$x^2 + 15^2 = 25^2$
$x^2 + 225 = 625$
$x^2 = 400$
$x = 20$

$20\frac{dx}{dt} = 25(-30)$

$\frac{dx}{dt} = \frac{25(-30)}{20} = -37.5$ ft./min.

3. Gravel is falling on a conical pile at the rate of $10\,\frac{ft^3}{min}$. At all times, the radius of the cone is twice the height of the cone. Find the rate of change of the height of the pile when the radius of the pile is 6 *ft*.



Key substitution: $r = 2h$

$V = \frac{1}{3}\pi r^2 h$ (Formula for the volume of a cone)

$V = \frac{1}{3}\pi (2h)^2 h$ substitution

$V = \frac{4}{3}\pi h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$ FIND

Since $r = 2h$, when $r = 6$
$6 = 2h$
$3 = h$

$10 = 4\pi (3)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{4\pi \cdot 9} = \frac{10}{36\pi} = \frac{5}{18\pi}$ ft./min.