LESSON 3-4 IMPLICIT DIFFERENTIATION

All the derivatives you have done to this point have been of explicit equations. For example $y = x^2$, $y = \frac{x}{x-1}$, and $y = \sqrt{2x+1}$ all explicitly express y in terms of x.

In this lesson you will be working with implicit equations where the relationship between x and y is only implied. $x^2 + y^2 = 1$, $xy + y^2 = 3$, and $xy = 1$ are all examples of implicit equations.

It is possible to differentiate implicit equations using **implicit differentiation**.

Procedure:
1. Differentiate both sides with respect to x. (Remember the y' "hook-on factor" for any term involving y.)
2. Collect all y' (or $\frac{dy}{dx}$) terms on one side of the equation.
3. Factor out y'.
4. Divide to solve for y'.

Remember: $\frac{dy}{dx} = y'$

Warm-up Examples: Differentiate.

1. $y = x$ $\quad \frac{dy}{dx} = 1$
2. $y = x^2$ $\quad \frac{dy}{dx} = 2x$
3. $y = (2x-1)^2$ $\quad \frac{dy}{dx} = 2(2x-1)^1(2)$
4. $y = (f(x))^2$ $\quad y' = 2f(x)\,f'(x)$
5. $x = y^2$ $\quad 1 = 2y\,y'$

Examples:

1. Given $x^2 - 2y^3 + 3x = 6$, find y'.

$2x - 6y^2y' + 3 = 0$

$2x + 3 = 6y^2y'$

$y' = \frac{2x+3}{6y^2}$

2. Find the slope of the lines tangent to and normal to the graph of $x^2 + 4y^2 = 25$ at $(3,2)$.

$2x + 8yy' = 0$

$8yy' = -2x$

$y' = \frac{-2x}{8y} = \frac{-x}{4y}$

$y'(3,2) = \frac{-3}{8} = m_{tan}$

$m_{norm} = \frac{8}{3}$

3. Given $x^3 - 2xy + y^3 = 5x$, find $\frac{dy}{dx}$ and evaluate at the point $(1,2)$.

Product

$3x^2 - (2xy' + y \cdot 2) + 3y^2y' = 5$

$3x^2 - 2xy' - 2y + 3y^2y' = 5$

$-2xy' + 3y^2y' = 5 - 3x^2 + 2y$

$y'(-2x + 3y^2) = 5 - 3x^2 + 2y$

$y' = \frac{5 - 3x^2 + 2y}{-2x + 3y^2}$

$y'(1,2) = \frac{5 - 3 + 4}{-2 + 12} = \frac{6}{10} = \frac{3}{5}$

4. Given $x^2 + y^2 = 3$, find y'' in terms of y.

$2x + 2yy' = 0$

$2yy' = -2x$

$y' = \frac{-2x}{2y} = \frac{-x}{y}$

$y'' = \frac{y(-1) - (-x)y'}{y^2}$

$= \frac{-y + xy'}{y^2}$

$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$ mult. by y / mult. by y

$= \frac{-y^2 - x^2}{y^3}$

$= \frac{-(y^2 + x^2)}{y^3}$

$= \frac{-3}{y^3}$