LESSON 3-3 THE CHAIN RULE AND THE GENERAL POWER RULE

General Power Rule: (used to differentiate a function of x raised to a power)
(a type of Chain Rule)

$\boxed{\frac{d}{dx}u^n = nu^{n-1}u'}$ (where u is a function of x)

Informally, we will call the u' factor the "hook-on factor". It is a crucial part of the chain rule.

Examples: Differentiate.

1. $f(x)=(x^2-3)^2$ $f'(x)=2(x^2-3)^1\cdot 2x$ Hook-on $=4x(x^2-3)$ $=4x^3-12x$ or $f(x)=x^4-6x^2+9$ $f'(x)=4x^3-12x$

2. $g(x)=\sqrt{(2x^2-x)^3}$ $g(x)=(2x^2-x)^{3/2}$ $g'(x)=\frac{3}{2}(2x^2-x)^{\frac{1}{2}}(4x-1)$ Hook on

3. $f(x)=3x^2\sqrt[3]{9-4x^2}=3x^2\cdot(9-4x^2)^{\frac{1}{3}}$ P.R: $f'(x)=3x^2\cdot\frac{1}{3}(9-4x^2)^{-2/3}(-8x)+(9-4x^2)^{\frac{1}{3}}(6x)$

Chain Rule: (used to differentiate any composition of functions)

$\frac{d}{dx}f(g(x))=f'(g(x))g'(x)$ This is not $f(x)\cdot g(x)$

Note: You must quickly learn to distinguish between the Chain Rule and the Product Rule!

4. If $p(2)=5$, $q(2)=3$, $q'(2)=2$, $p'(3)=4$, and $q'(3)=\frac{3}{2}$, find $\frac{d}{dx}p(q(x))$ at $x=2$.

$\frac{d}{dx}p(q(x))=p'(q(x))\cdot q'(x)$

(at $x=2$) $=p'(q(2))\cdot q'(2)=p'(3)\cdot 2=4\cdot 2=8$

ASSIGNMENT 2-5

Find the derivative.

1. $y=(3x+5)^3$
2. $f(x)=3(7x+5)^4$
3. $y=\sqrt{2-3x}$
4. $f(t)=\frac{1}{(1-t)^2}$
5. $y=\sqrt[3]{(x^2+1)^2}$
6. $g(x)=x(2x+3)^3$
7. $y=\frac{1}{\sqrt{x+1}}$
8. $f(x)=\frac{3x-2}{x+1}$

Find an equation of the line tangent to the graph of f at the given point.

9. $f(x)=\sqrt{2x^2+2}$ at $(-1,2)$
10. $f(x)=\frac{x+4}{x}$ at $(2,3)$

Find the indicated derivatives.

11. $\frac{d}{dx}(2x-3)^4$
12. $\frac{d^2}{dt^2}(t^2-1)^{\frac{3}{2}}$

13. Find the point(s) at which a line tangent to the graph of $f(x)=(2x-3)^3$ is parallel to the graph of $y=24x-7$. You may use a calculator.

14. If $g(x)=(f(x))^3$, $f(1)=2$, and $f'(1)=4$, find $g'(1)$.