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LESSON 3-1 POSITION, VELOCITY, ACCELERATION

Important Terms	
Position Function	gives the location of an object at time t, usually $s(t)$, $x(t)$, or $y(t)$
Velocity Function	the rate of change (derivative) of position, usually $v(t)$ Velocity is positive for upward or rightward motion and negative for downward or leftward motion.
Acceleration Function	the rate of change (derivative) of velocity, usually $a(t)$
Initial Position	starting position (at $t = 0$), s_o
Initial Velocity	starting velocity (at $t = 0$), v_o
Speed	the absolute value of velocity
Displacement	the net change in position, (final pos. – original pos.)
Total Distance	total distance traveled by the object in the time interval (takes into account all direction changes)

Example 1. If $s(t) = t^3 + t$, find $v(t)$ and $a(t)$.

$v(t) = 3t^2 + 1$

$a(t) = 6t$

Examples: Use the position function $s(t) = 16t^3 - 36t^2 + 24$ of an object moving on a horizontal line for Examples 2-11. Distance units are measured in feet and time units are measured in seconds.

2. What is the initial position of the object?

$s(0) = 24$ ft.

3. What is the velocity of the object at $t = 1$ second?

$v(t) = 48t^2 - 72t$

$v(1) = 48 - 72$

$= -24$ ft./sec.

4. What is the speed of the object at $t = 1$ second?

$|v(1)| = 24$ ft./sec.

5. What is the acceleration of the object at $t = 1$ second?

$a(t) = 96t - 72$

$a(1) = 96 - 72 = 24$ ft./sec./sec.

or 24 ft./sec^2

6. When is the object at rest?

$v(t) = 0$

$48t^2 - 72t = 0$

$24t(2t - 3) = 0$

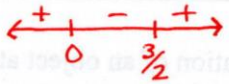
$t = 0$ sec., $t = \frac{3}{2}$ sec.

7. When is the object moving right?

$v(t) > 0$

$t < 0$ sec.

$t > \frac{3}{2}$ sec.



8. When is the object moving left?

$v(t) < 0$

$0 < t < \frac{3}{2}$ sec.

9. When is the velocity of the object equal to 54 $\frac{ft}{sec}$?

$48t^2 - 72t = 54$

Use calculator: $Y_1 = Y_2$

$t = -.549$ sec,

$t = 2.049$ sec.

10. What is the displacement of the object between $t = 0$ and $t = 2$ seconds?

$s(2) - s(0) = 16(8) - 36(4) + 24 - 24$

$= 128 - 144 = -16$ ft.

(means 16 ft. to the left of the initial position)

11. What is the total distance traveled by the object between $t = 0$ and $t = 2$ seconds?

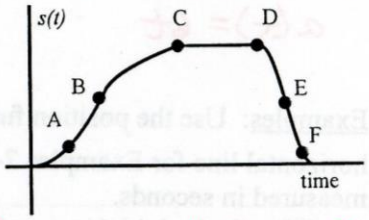
start: $s(0) = 24$

position change: $s(\frac{3}{2}) = -3$ $\quad 24 - (-3) = 27$ ft. (to the left)

end: $s(2) = 8$ $\quad 8 - (-3) = 11$ ft. (to the right)

T.D. $= 27 + 11 = 38$ ft.

The graph shows the position function of a radio controlled model car. Answer these questions and explain.



12. Was the car going faster at A or at B? B

The curve is "steeper" at point B.

13. When was the car stopped? Between C and D.

The velocity (slope of the position curve) is zero between C and D.

14. At which point was the car's velocity the greatest? B

The slope of the position graph is greatest at Point B.

15. At which point was the car's speed the greatest? E

The absolute value of the slope of the position graph is greatest at Point E.

Vertical Motion Examples:

Suppose $s(t) = -16t^2 + 48t + 160$ gives the position (in feet) above the ground for a ball thrown into the air from the top of a high cliff (where time is measured in seconds).

16. Find the initial velocity.

$v(t) = -32t + 48$

$v(0) = 48$ ft./sec.

17. At what time does the ball hit the ground?

$s(t) = 0$

$-16(t^2 - 3t - 10) = 0$

$-16(t - 5)(t + 2) = 0$

$t = 5$ or $\cancel{-2}$

$t = 5$ sec.

18. At what time does the ball reach its maximum height?

$v(t) = 0$

$-32t + 48 = 0$

$48 = 32t$

$t = \frac{48}{32} = 1.5$ sec.