LESSON 2-5 DERIVATIVE RULES (short cuts). TANGENT LINES. NONDIFFERENTIABILITY. RATE OF CHANGE

The expression $\frac{d}{dx}$ (pronounced dee, dee x) means to differentiate with respect to x. The most common derivative $\frac{d}{dx}y$ (dee, dee x of y) is usually written as $\frac{dy}{dx}$.

Derivative Rules:

Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$

Constant Rule: If c is any constant, $\frac{d}{dx}c = 0$.

Scalar Multiple Rule: If c is any constant, $\frac{d}{dx}\left(c\,f(x)\right) = c\,f'(x)$.

Sum Rule: $\frac{d}{dx}\left(f(x)+g(x)\right) = f'(x)+g'(x)$

Examples: Differentiate.

1. $f(x) = x^4$ 2. $y = x^{-\frac{2}{3}} + 3$ 3. $h(t) = 5 - \frac{1}{2t^3}$ 4. $f(x) = \frac{5}{(2x)^3}$

$f'(x) = 4x^3$ $y' = -\frac{2}{3}x^{-5/3}$ $h(t) = 5 - \frac{1}{2}t^{-3}$ $f(x) = \frac{5}{8x^3} = \frac{5}{8}x^{-3}$

$h'(t) = \frac{3}{2}t^{-4}$ $f'(x) = -\frac{15}{8}x^{-4}$

Higher-Order Derivatives

Since the derivative of a function is another function, we can repeat the differentiation process to find the derivative of a derivative. The result is still another function which could again be differentiated. These derivatives are called higher-order derivatives.

Notation:

First Derivative:	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}f(x)$
Second Derivative:	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}f(x)$
Third Derivative:	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}f(x)$
Fourth Derivative:	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}f(x)$

Example 5. For $f(x) = \frac{1}{2\sqrt[3]{x^2}}$, find $f'(1)$ and $f''(-8)$.

$f(x) = \frac{1}{2}x^{-2/3}$ $f''(x) = \frac{5}{9}x^{-8/3}$

$f'(x) = -\frac{1}{3}x^{-5/3}$ $f''(-8) = \frac{5}{9}(-8)^{-8/3}$

$f'(1) = -\frac{1}{3}(1)^{-5/3} = -\frac{1}{3}$

Equation of a Tangent Line:

Since the derivative of a function gives us a slope formula for tangent lines to the graph of the function, the derivative can be used to find equations of tangent lines.

Sometimes we will want to find a line perpendicular to the tangent line at a certain point. Such a line is called a normal line.

$$m_{\text{normal}} = \frac{-1}{m_{\text{tangent}}}$$

Examples:

6. Find an equation of the line tangent to the graph of $f(x) = 4x^5 - 3x^2 + 5$ at the point (1,6).

$f'(x) = 20x^4 - 6x$

$f'(1) = 20 - 6 = 14$

$m_{tan.} = 14$

T.L: $y - 6 = 14(x - 1)$

need point & slope. slope must be a number.

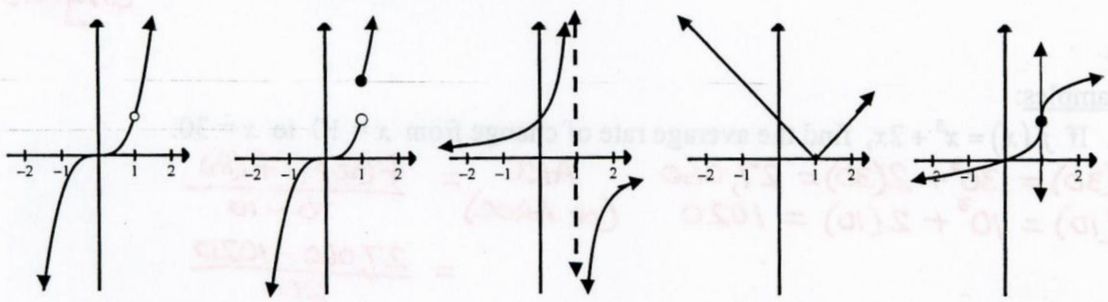
7. Find an equation of the normal line to the same curve at the same point.

$m_{norm.} = -\frac{1}{14}$

N.L: $y - 6 = -\frac{1}{14}(x - 1)$

NONDIFFERENTIABILITY (when a derivative does not exist)

Each of these functions has no derivative when $x = 1$.



hole | jump | vertical asymptote | sharp turn | vertical tangent

These five characteristics destroy differentiability:

1. Holes
2. Jumps (breaks)
3. Vert. Asymptotes

} discontinuities

4. Sharp Turns
5. Vert. Tangent Lines

Note:
If a function is not continuous, it is not differentiable (see the first three figures above). A function may be continuous and still not be differentiable (see the last two figures above).

Examples: Find the x-values where $f(x)$ is not differentiable. Give a reason for each.

8. $f(x)=|x|$

$x=0$
sharp turn

9. $f(x)=\begin{cases} x^2, & x\le 0 \\ x, & x>0 \end{cases}$

$f'(x)=\begin{cases} 2x, & x<0 \\ 1, & x>0 \end{cases}$

$x=0$ sharp turn

10. $f(x)=\begin{cases} x^2, & x\le 0 \\ x^2+1, & x>0 \end{cases}$

$x=0$
not continuous (jump)

11. $f(x)=\dfrac{x}{x(x-1)}$

$x=0$ (hole)
not continuous
$x=1$ (V.A.)
not continuous

12. $f(x)=\sqrt[3]{x} = x^{1/3}$

$f'(x)=\frac{1}{3}x^{-2/3} = \dfrac{1}{3\sqrt[3]{x^2}}$

$x=0$
vertical tangent
or sharp turn

Rate of Change:

Another meaning for slope is rate of change. We now have two ways to find slopes (rates of change).

1. **Average Rate of Change** This is the slope between two points. It is found without using a derivative (algebraically). $\text{ARC} = m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2-y_1}{x_2-x_1}$

slope of secant line

2. **Instantaneous Rate of Change** This is the slope at a single point. It is usually found by using a derivative (calculus). $\text{IRC} = m = f'(c)$

slope of tangent line

Examples:

13. If $f(x)=x^3+2x$, find the average rate of change from $x=10$ to $x=30$.

$f(30) = 30^3+2(30) = 27{,}060$
$f(10) = 10^3+2(10) = 1020$

ARC (or AROC) $= \dfrac{f(30)-f(10)}{30-10} = \dfrac{27{,}060-1020}{20} = 1302$

14. If $f(x)=x^3+2x$, find the instantaneous rate of change when $x=10$.

$f'(x)=3x^2+2$
$f'(10)=3(10)^2+2=302=$ IRC (or IROC)