

LESSON 2-3 CURVE SKETCHING

What to watch for:

Domain: possible x -values. Avoid zero denominators and square roots of negative numbers.

Vertical Asymptotes: denominator restrictions from the **reduced** function (write equations in the form $x = a$)

Holes: denominator restrictions from the **original** function which are no longer restricted in the reduced function (plug into the reduced function to find the y -value and write as ordered pairs)

x -intercepts: let $y = 0$, solve for x (write as ordered pairs)

y -intercepts: let $x = 0$, solve for y (write as ordered pairs)

End Behavior: look at highest degree terms in the numerator and the denominator, analyze for "large" positive and negative x -values

Even/Odd: for vertical asymptotes and x -intercepts (or holes on the x -axis)

Note: These come from degrees of factors in the reduced function.

Examples: For Examples 1-3 give the domain, reduce the function, find vertical asymptotes, holes, and end behavior.

$$1. f(x) = \frac{x+2}{x^2-2x}$$

Do: $x \neq 0, 2$

V.A.: $x = 0, x = 2$ Both odd

E.B.: $y = \frac{1}{x}$

As $x \rightarrow \pm\infty, y \rightarrow 0$ (x -axis)

H.A.: $y = 0$

$$2. g(x) = \frac{2x^3}{(x+3)^2} = \frac{2x^3}{x^2 \dots}$$

Do: $x \neq -3$

V.A.: $x = -3$ even

E.B.: $y = 2x$ ↑

↓

$$3. h(x) = \frac{(x+4)(x-2)}{x^2+2x-8}$$

$$h(x) = \frac{(x+4)(x-2)}{(x+2)(x-2)}$$

Do: $x \neq \pm 2$

$h_{red}(x) = \frac{x+4}{x+2}$

V.A.: $x = -2$ odd

Hole: $(2, \frac{5}{4})$ or $(2, \frac{3}{2})$

E.B.: $y = 1$ H.A.

Curve Sketching Recipe:

1. Give the domain.
2. Reduce $f(x)$. Oftentimes, you must factor before you can reduce.
3. Find vertical asymptotes and holes.
4. Give x - and y -intercepts.
5. Find the end behavior (horizontal asymptotes or other).
6. (optional) Check for symmetry.
7. (if needed) Find a starting point.
8. Graph.

Examples: Follow the **Curve Sketching Recipe** to graph.

4. $f(x) = x(x-1)(x+2)^2$

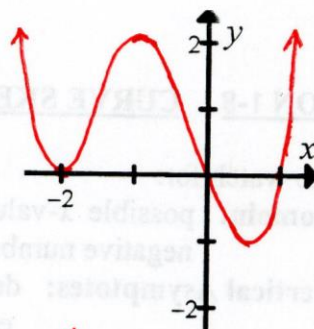
Do: **all reals**

y-int.: $(0,0)$

x-int.: $(0,0)$, $(1,0)$, $(-2,0)$

E.B.: $y = x^4$

odd odd even
↑ ↑



5. $g(x) = \frac{x(x-1)^2(x+3)^3}{x^2(x-1)(x-3)^2}$

Do: $x \neq 0, 1, 3$

$$g_{red}(x) = \frac{(x-1)^1(x+3)^3}{x^1(x-3)^2}$$

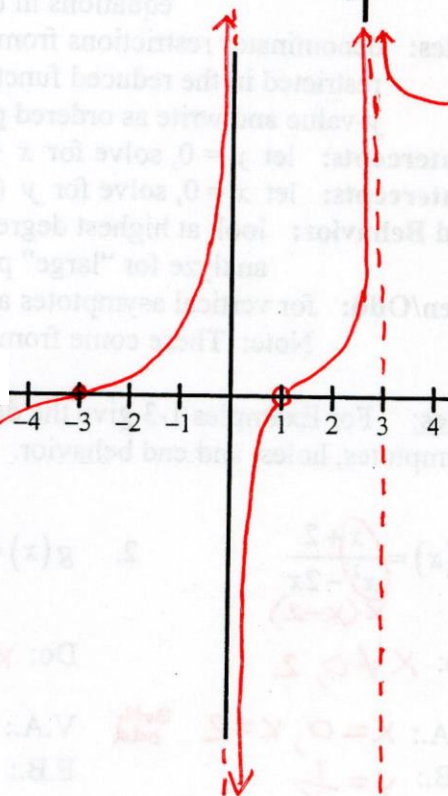
V.A.: $x=0$, $x=3$
odd, even

Holes: $(1,0)$ odd

x-int.: $(-3,0)$ odd

y-int.: none (can't use $x=0$)

E.B.: $y = \frac{x^4 \dots}{x^3 \dots} = x$



6. $y = \frac{x+1}{\sqrt{x^2}}$ (X)

Do: $x \neq 0$

Holes: none

y-int.: none

Starting Point: $(1, 2)$

↑
you choose

V.A.: $x=0$

x-int.: $(-1, 0)$ odd

E.B.: Right: $y = \frac{x}{|x|}$

H.A.: $y = 1$

Left: $y = \frac{x}{|x|}$

H.A.: $y = -1$

