LESSON 2-2 INFINITE LIMITS, LIMITS AT INFINITY

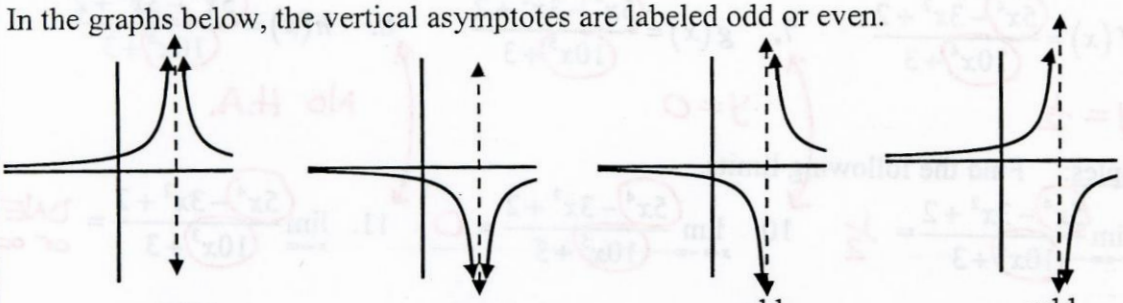
Review:

The graph of the function $f(x)=\dfrac{x-1}{(x-1)(x-2)^2(x-4)^3}$ has a **hole** at $x=$ 1,

an **even vertical asymptote** at $x=$ 2,

and an **odd vertical asymptote** at $x=$ 4.

In the graphs below, the vertical asymptotes are labeled odd or even.



even even odd odd

Infinite Limits use sign analysis

You have seen examples where a limit does not exist at a vertical asymptote. Such non-existent limits can be expressed as infinite limits if the vertical asymptote is even or if you are finding one-sided limits. We will write $\lim\limits_{x\to c} f(x)=\infty$ or $\lim\limits_{x\to c} f(x)=-\infty$.

The examples below make use of your knowledge of even and odd vertical asymptotes as well as holes.

Examples:

1. $\lim\limits_{x\to 2^+}\dfrac{x+3}{x-2}=\infty$ (5 / 0; $\frac{+}{+}$; ODD V.A.)

2. $\lim\limits_{x\to 2^-}\dfrac{x+3}{x-2}=-\infty$ (5 / 0; $\frac{+}{-}$)

3. $\lim\limits_{x\to 1^+}\dfrac{x-2}{(x-1)^2}=-\infty$ (−1 / 0; $\frac{-}{+}$)

4. $\lim\limits_{x\to 1^-}\dfrac{x-2}{(x-1)^2}=-\infty$ (−1 / 0; $\frac{-}{+}$; EVEN V.A.)

5. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2}=$ (0 / 0) $\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)}{x-2}=\lim\limits_{x\to 2}(x+2)=4$ (hole in the graph at (2,4))

Limits at Infinity

If the graph of a function $f(x)$ approaches a horizontal asymptote to the left and/or the right, $f(x)$ is said to have a limit at infinity. If the asymptote is $y = L$ then $\lim_{x\to\infty} f(x) = L$. In other words, limits at infinity give us end behaviors for graphs of functions. For "large" values of x, the highest degree terms in the numerator and denominator dominate the other terms and are the only terms you need to consider.

Review Examples: Find the horizontal asymptotes.

6. $f(x) = \dfrac{5x^4 - 3x^2 + 2}{10x^4 + 3}$ $y = \frac{1}{2}$

7. $g(x) = \dfrac{5x^4 - 3x^2 + 2}{10x^5 + 3}$ $y = 0$

8. $h(x) = \dfrac{5x^4 - 3x^2 + 2}{10x^3 + 3}$ No H.A.

Examples: Find the following limits.

9. $\lim_{x\to\infty} \dfrac{5x^4 - 3x^2 + 2}{10x^4 + 3} = \frac{1}{2}$

10. $\lim_{x\to-\infty} \dfrac{5x^4 - 3x^2 + 2}{10x^5 + 3} = 0$

11. $\lim_{x\to\infty} \dfrac{5x^4 - 3x^2 + 2}{10x^3 + 3} =$ DNE or ∞

12. $\lim_{x\to\infty} \dfrac{(2x+3)(x-1)^2}{(x+2)(3x-1)^2} =$

Note: Make sure you consider highest degree **terms** – not highest degree **factors.**

$\lim_{x\to\infty} \dfrac{2x^3 \ldots}{9x^3 \ldots} = \frac{2}{9}$

Rational functions like those above have at most one horizontal asymptote, so the limit is the same whether x approaches ∞ or $-\infty$. However, radical functions frequently have two horizontal asymptotes.

Examples: Find these limits.

13. $\lim_{x\to\infty} \dfrac{\sqrt{4x^2 - 3}}{x} =$

$\lim_{x\to\infty} \dfrac{|2x|}{x} = 2$ $\frac{+}{+}$

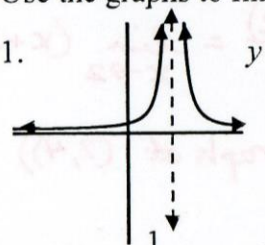
14. $\lim_{x\to-\infty} \dfrac{\sqrt{4x^2 - 3}}{x} =$

$\lim_{x\to-\infty} \dfrac{|2x|}{x} = -2$ $\frac{+}{-}$

ASSIGNMENT 2-2

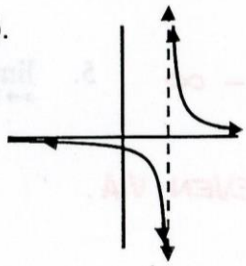
Use the graphs to find these limits (answer ∞ or $-\infty$).

1. $y = \dfrac{1}{(x-2)^2}$



a. $\lim_{x\to2^-} \dfrac{1}{(x-2)^2}$ b. $\lim_{x\to2^+} \dfrac{1}{(x-2)^2}$

2. $y = \dfrac{1}{x-2}$



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