LESSON 2-1 MORE LIMITS, MORE CONTINUITY, INTERMEDIATE VALUE THEOREM

If direct substitution does not give an answer to a limit problem because an indeterminate form is obtained (usually $\frac{0}{0}$), use algebraic techniques to change the form of the limit.

Examples:

1. $\lim_{x \to 0} \frac{x}{x(x+1)}$ $\frac{0}{0}$

$= \lim_{x \to 0} \frac{1}{x+1} = 1$

* (hole at (0,1))

2. $\lim_{x \to 2} \frac{x-2}{x^2-4}$ $\frac{0}{0}$

$= \lim_{x \to 2} \frac{(x-2)}{(x+2)(x-2)}$

$= \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{4}$

(hole at $(2, \frac{1}{4})$)

3. $\lim_{x \to 1} \frac{x^3-1}{x-1}$ $\frac{0}{0}$ $= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$= \lim_{x \to 1} (x^2+x+1) = 3$

(hole at (1,3))

$$\begin{array}{r} x^2+x+1 \\ x-1 \overline{)\, x^3 \qquad\quad -1} \\ -x^3+x^2 \qquad\quad \\ x^2 \qquad \\ -x^2+x \qquad \\ x-1 \\ -x+1 \\ 0 \end{array}$$

4. $\lim_{x \to 1} \frac{(\sqrt{x}-1)}{(x-1)} \cdot \frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$ $\frac{0}{0}$

$= \lim_{x \to 1} \frac{x-1}{(x-1)(\sqrt{x}+1)}$

$= \lim_{x \to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{2}$

(hole at $(1, \frac{1}{2})$)

5. $\lim_{x \to 2^+} \frac{x}{x-2}$ $\frac{2}{0}$ DNE or $+\infty$

$\frac{+}{+}$ (sign analysis)

V.A. at $x=2$

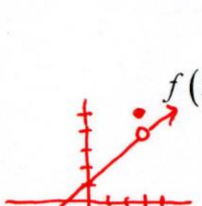
6. Discuss the continuity of

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$$

$\frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}}$

Hole at (3, 4)

Point at (3,5)



Removable discontinuity at $x = 3$

7. If $g(x) = \begin{cases} 3x^2+a, & x > 2 \\ x-3, & x \le 2 \end{cases}$ is a continuous function, find the value of a.

$3(2)^2 + a = 2-3$

$12 + a = -1$

$a = -13$

8. Use a calculator to find these limits.

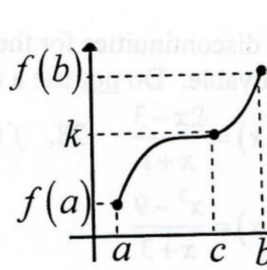
a. $\lim_{x \to 1} \frac{x^3-3x^2+x+2}{x^3-2x^2-x+2}$

DNE

b. $\lim_{x \to 2} \frac{x^3-3x^2+x+2}{x^3-2x^2-x+2}$

.333

Intermediate Value Theorem



① ②

If f is continuous on $[a,b]$ and k is any y-value between $f(a)$ and $f(b)$, then there is at least one x-value c between a and b such that $f(c)=k$.

③

Examples:

9. Does the Intermediate Value Theorem guarantee a c-value on the given interval?

a. $f(x)=x^2-x$, $f(c)=12$, $[0,5]$

① $f(x)$ is continuous on $[0,5]$.

② $f(0)=0$, $f(5)=20$, and $0<12<20$

③ IVT guarantees a c-value in $[0,5]$ such that $f(c)=12$.

b. $g(x)=\dfrac{x^2-4}{x-2}$, $g(c)=4$, $[0,3]$

① $g(x)$ is not continuous on $[0,3]$

IVT does not apply

10. Find the value of c in Example 9a.

Thought: $f(c)=k$, so $c^2-c=12$

(or $x^2-x=12$)

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x=4$ or $x=\cancel{-3}$ (not in the interval)

$\boxed{c=4}$ is the value in the interval $[0,5]$

ASSIGNMENT 2-1

Find the indicated limits without using a calculator. **Show steps using correct limit symbolism!**

1. $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}$
2. $\lim\limits_{x\to -1}\dfrac{2x^2-x-3}{x+1}$
3. $\lim\limits_{x\to -1}\dfrac{x^2-1}{x-1}$
4. $\lim\limits_{x\to -1}\dfrac{x^3+1}{x+1}$
5. $\lim\limits_{x\to 5^+}\dfrac{x-5}{x^2-25}$
6. $\lim\limits_{x\to -5}\dfrac{x-5}{x^2-25}$
7. $\lim\limits_{x\to 2}\dfrac{2-x}{x^2-4}$
8. $\lim\limits_{x\to 3}\dfrac{x^2-9}{x^2-6x+9}$
9. $\lim\limits_{x\to 2^-}\dfrac{1}{x^2-4}$
10. $\lim\limits_{x\to 1}\dfrac{x}{x^2+1}$
11. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{x-4}$
12. $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{x+1}-2}$
13. $\lim\limits_{x\to 0^-}\dfrac{|x|}{x}$
14. $\lim\limits_{x\to 0}\dfrac{|x|}{x}$
15. $\lim\limits_{x\to 3^+}[\![x-1]\!]$
16. $\lim\limits_{x\to 3^-}[\![x-1]\!]$
17. $\lim\limits_{x\to 3}[\![x-1]\!]$
18. $\lim\limits_{x\to 2}[\![x+6]\!]$
19. $\lim\limits_{x\to 3}\left[\!\left[\dfrac{x}{2}\right]\!\right]$
20. $\lim\limits_{x\to 5^-}[\![2x-3]\!]$
21. $\lim\limits_{x\to 3}\begin{cases}\frac{1}{2}x+1, & x\le 3\\ \dfrac{12-2x}{3}, & x>3\end{cases}$
22. $\lim\limits_{x\to 1}\begin{cases}x^2+1, & x<1\\ x^3+1, & x\ge 1\end{cases}$
23. $\lim\limits_{x\to 2}\begin{cases}x-2, & x\le 0\\ x+2, & x>0\end{cases}$

Use a calculator to find these limits.

24. (a) $\lim\limits_{x\to 1}\dfrac{\sin x}{6x}$ (b) $\lim\limits_{x\to 0}\dfrac{\sin x}{6x}$
25. $\lim\limits_{x\to -2}\dfrac{x^3+2x^2-x-2}{x^3+2x^2+x+2}$
26. $\lim\limits_{x\to 2}\dfrac{|2-x|}{25x-50}$