LESSON 1-4 LIMITS , CONTINUITY

Limits

Informally, a **limit is a *y*-value** which a function approaches as x approaches some value.

$\lim_{x\to c} f(x) = L$ means as x approaches c, $f(x)$ approaches the y-value of L.

Examples

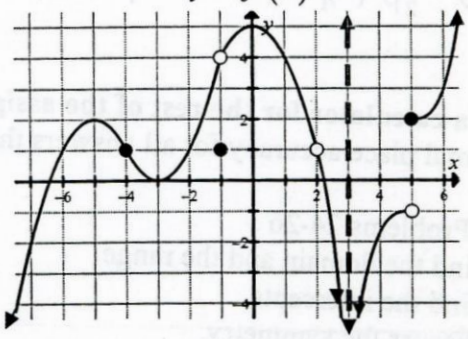
limits:

1. $\lim_{x\to -4} f(x) =$ 1
2. $\lim_{x\to -1} f(x) =$ 4
3. $\lim_{x\to 2} f(x) =$ 1
4. $\lim_{x\to 3} f(x) =$ $-\infty$ or DNE
5. $\lim_{x\to 5} f(x) =$ DNE

function values:

8. $f(-4) =$ 1
9. $f(-1) =$ 1
10. $f(2) =$ DNE
11. $f(3) =$ DNE
12. $f(5) =$ 2

$y = f(x)$



one-sided limits:

6. $\lim_{x\to 5^-} f(x) = -1$
7. $\lim_{x\to 5^+} f(x) =$ 2

Continuity

Informally, a function is continuous where it can be drawn without lifting a pencil.
Roughly, continuous means "connected."
Formally, a function is continuous where its limit and function value are the same.
In this course, we will work with three types of discontinuities: holes, vertical asymptotes, and jumps (breaks).

Example 13. List the x-values of the discontinuities of the function $y = f(x)$ graphed above. $x = -1, 2, 3, 5$

All discontinuities can be classified as removable or nonremovable.
Removable discontinuities occur when the function has a limit (holes in the graph).
Nonremovable discontinuities occur when the limit of the function does not exist (jumps and vertical asymptotes).

Example 14. Which of the discontinuities from Example 13 are removable? $x = -1, 2$

At x-values where a function is continuous, limits can be found by **direct substitution**.

Examples:

15. $\lim_{x\to 3}(3x^2+2) =$ $3(3)^2+2$ or 29
16. $\lim_{x\to 1} \frac{x^2+x}{x+1} =$ $\frac{1^2+1}{1+1}$ or 1

For **piecewise functions**, one-sided limit evaluation is often necessary.

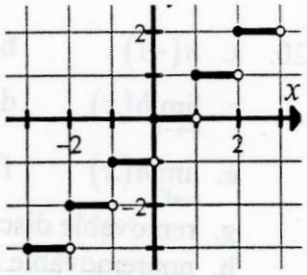
Examples:

17. If $f(x)=\begin{cases} 4-x, & x\le 1 \\ 4x-x^2, & x>1 \end{cases}$, $\lim_{x\to 1} f(x) =$ 3 $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 3$

18. If $g(x)=\begin{cases} 3x-x^3, & x\le 1 \\ 2x^2-1, & x>1 \end{cases}$, $\lim_{x\to 1} g(x) =$ DNE $\lim_{x\to 1^-} g(x) = 2$ $\lim_{x\to 1^+} g(x) = 1$

19. For this same g function, $\lim_{x\to -1} g(x) =$ $3(-1)-(-1)^3$ or $-3+1$ or -2

Another function requiring one-sided limit analysis is a step function called the **Greatest Integer Function.** $f(x)=[\![x]\!]=$ the greatest integer less than or equal to x. The graph is shown at the right.



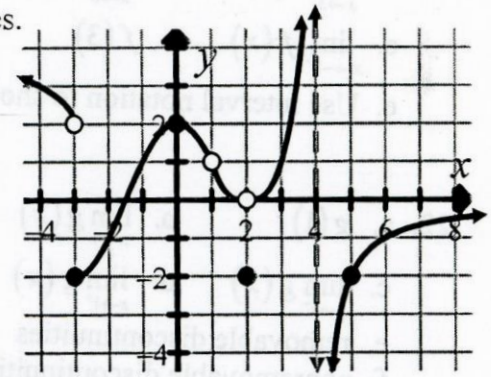
Examples: Find the following limits.

20. $\lim_{x\to \frac{1}{2}} [\![x]\!] =$ 0

21. $\lim_{x\to 1} [\![x]\!] =$ DNE

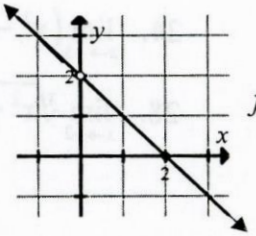
ASSIGNMENT 1-4

Use the graph of $y=f(x)$ at the right to find these values.



1. $\lim_{x\to 5} f(x)$
2. $\lim_{x\to -3} f(x)$
3. $\lim_{x\to -3^-} f(x)$
4. $\lim_{x\to -3^+} f(x)$
5. $f(-3)$
6. $\lim_{x\to 4^-} f(x)$
7. $\lim_{x\to 0} f(x)$
8. $f(0)$
9. $\lim_{x\to 4} f(x)$
10. $\lim_{x\to 4^+} f(x)$
11. $f(4)$
12. $f(2)$
13. $\lim_{x\to 2} f(x)$
14. $f(1)$
15. $\lim_{x\to 1} f(x)$
16. List the x-values of all removable discontinuities of $f(x)$.
17. List the x-values of all nonremovable discontinuities of $f(x)$.

Use the graph shown to find each value.



$f(x)=\dfrac{-2x^2+4x}{2x}$

18. a. $\lim_{x\to 0} f(x)$ b. $\lim_{x\to 2} f(x)$
    c. $f(0)$ d. removable discontinuities