LESSON 1-3 INTERCEPTS, SYMMETRY, EVEN/ODD, INTERSECTIONS

x- and y-intercepts

x-intercepts are points where a graph crosses or touches the x-axis. The y-coordinate is zero. To find the x-intercept, let $y = 0$ and solve for x.

y-intercepts are points where a graph crosses or touches the y-axis. The x-coordinate is zero. To find the y-intercept, let $x = 0$ and solve for y.

Example 1.

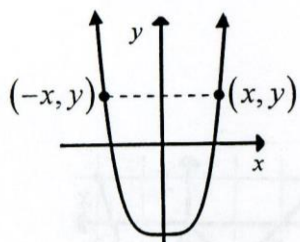
Find the x- and y-intercepts for $y^2 - 3 = x$.

X-int: $0 - 3 = x$ y-int: $y^2 - 3 = 0$ $y^2 = 3$

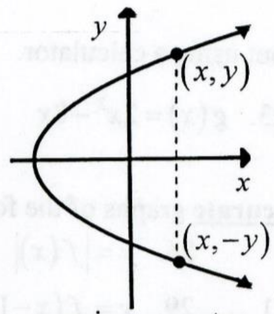
Thought: Let $y=0$ $(-3,0)$ Thought: Let $x=0$ $y = \pm\sqrt{3}$

$(0, \pm\sqrt{3})$

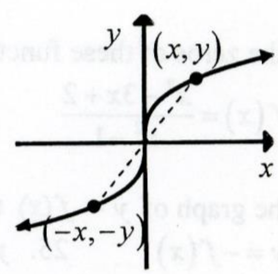
Symmetry



y-axis symmetry
reflection across the y-axis



x-axis symmetry
reflection across the x-axis



origin symmetry
reflection through the origin (0,0)

Graphs can be symmetric to other lines and points. However, we will concentrate on these three.

Formal tests for symmetry:

1. y-axis: replacing x with $-x$ produces an equivalent equation
2. x-axis: replacing y with $-y$ produces an equivalent equation
3. origin: replacing x with $-x$ and y with $-y$ produces an equivalent equation

Informal tests for symmetry:

1. y-axis: substituting a number and its opposite for x give the same y-value
2. x-axis: substituting a number and its opposite for y give the same x-value
3. origin: substituting a number and its opposite for x give opposite y-values

Note: These informal tests are not foolproof. Think about whether other numbers would work the same. If your substitution produces zero, try another number.

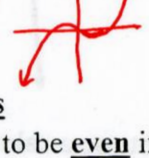
Examples: Find the type(s) of symmetry for the graph of:

2. $y = 2x^3 - x$ 3. $y = |x| - 2$ 4. $|y| = x - 2$

Informal: If $x=1$, $y=1$; If $x=-1$, $y=-1$

Formal: $2(-x)^3 - (-x)$
$= -2x^3 + x = -y$

origin symmetry



If $x=1$, $y=-1$; If $x=-1$, $y=-1$

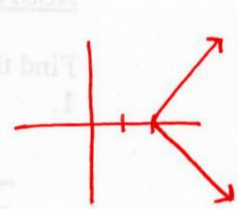
or $|-x| - 2 = |x| - 2$

or

y-axis symmetry

If $x=3$, $y=\pm 1$
(opposite y-values produce the same x-value)

or graph



x-axis symmetry

NOT a function

Even/Odd Functions

A function is defined to be **even** if $f(-x) = f(x)$ for all x in the domain of $f(x)$. Even functions have graphs with y-axis symmetry. Examples: $y = x^2$, $y = x^4$, $y = x^2 + 3$, $y = x^4 + x^2$ How about $y = |x^3|$?

A function is defined to be **odd** if $f(-x) = -f(x)$ for all x in the domain of $f(x)$. Odd functions have graphs with origin symmetry. Examples: $y = x$, $y = x^3$, $y = x^5$, $y = x^5 - x^3$

Examples: Determine whether the following functions are even, odd, or neither.

5. $f(x) = x^3 - x$ 6. $g(x) = x^2 - 4$ 7. $h(x) = x^2 + 2x + 2$

$f(2) = 8 - 2 = 6$
$f(-2) = -8 + 2 = -6$
ODD
(origin sym.)

$g(1) = -3$
$g(-1) = -3$
EVEN
(y-axis sym.)

$h(1) = 5$
$h(-1) = 1$
NEITHER

Points of Intersection of Two Graphs (without a calculator)

Method 1. Solve one equation for one variable and substitute into the other equation.

Method 2. Solve both equations for the same variable and set equal.

Example 8. Without using a calculator, find all points of intersection for the graphs of $x - y = 1$ and $x^2 - y = 3$.

$y = x - 1$ $y = x^2 - 3$

METHOD 1
Substitute:
$x^2 - (x-1) = 3$
$x^2 - x + 1 = 3$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2, -1$
$\boxed{(2,1) \quad (-1,-2)}$

METHOD 2
Since $y = x - 1$ and $y = x^2 - 3$
$x - 1 = x^2 - 3$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$
$x = 2, -1$
$\boxed{(2,1) \quad (-1,-2)}$

METHOD 3 (Elimination)
$-x + y = -1$
$x^2 - y = 3$
$x^2 - x = 2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2, -1$
$\boxed{(2,1) \quad (-1,-2)}$